HANDOUT KEY

Replace the Script

"I, We, You"

with Algebrafied Problems

A man twice as old as his sister weighs 10 pounds more than his cousin and has 12 more dimes than quarters in his pocket. He rows 5 miles upstream to get to a candy store. When he left, a plane left for New York flying against a 50 MPH headwind. He uses the money to buy mixed nuts, of which 10 percent are cashews. How soon after the trains meet does he arrive at the store? What are the dimensions of the store?

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Effective Mathematics Teaching Practices

1. Establish mathematics **goals** to focus learning.
2. Implement **tasks** that promote reasoning and problem solving.
3. Use and connect mathematical **representations.**
4. Facilitate meaningful mathematical **discourse**.
5. Pose purposeful **questions**.
6. Build **procedural fluency** from conceptual understanding.
7. Support **productive struggle** in learning mathematics.
8. **Elicit and use evidence** of student thinking.

NCTM NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS

NAME ______________________

Candy Jar Problem

Suppose you have a new candy jar with the same ratio of Jolly Ranchers (JR) to jawbreakers (JB) as shown in the picture, but it contains 100 Jolly Ranchers.

How many jawbreakers do you have?

Justify your answer.

Note: In the picture, Jolly Ranchers are represented by 5 rectangles, and jawbreakers are shown by 13 circles.

Fig. 12. The Candy Jar task. Adapted from Smith et al. (2005).

TABLE

JAR #	# JR's	# JB's
1	5	13
2	10	26
20	100	260

RATIOS

$\frac{JR}{JB}$ $\frac{5}{13}, \frac{10}{26}, \frac{15}{39}, \ldots \frac{100}{260}$

PROPORTION

$\frac{JR}{JB}$ $\frac{5}{13} = \frac{100}{x}$

SCALE FACTOR

$\frac{5}{13} \overset{\cdot 20}{=} \frac{100}{x}$

DIRECT VARIATION

$100 = 5k$

$20 = k$

$\# JB = 13k$

$= 13 \cdot 20$

$= 260$

SUMMARY OF TECHNIQUES

- Table
- Look for pattern
- Write out equal ratios / fractions
- Solve proportion
- Make direct variation
- Rate of change, make graph
- Make graph - discrete points, 5 right, 13 up

National Council of Teachers of Mathematics (NCTM). *Principles to Actions*. Reston, Va.: NCTM, 2014.

Fun Tees
Version 1

Fun Tees is offering a 30% discount on all merchandise. Find the amount of discount on a T-shirt that was originally priced at $16.00.

30% of 16 $\frac{30}{100} = \frac{x}{16}$

$4.80 $x = 4.80$

Fun Tees
Version 2

Fun Tees is offering a 30% discount on all merchandise.

a. Find the amount of discount on a T-shirt that was originally priced at $16.00. $4.80

b. Suppose the T-shirt was originally priced at $17, $18, 19, 20, or $50. Describe the amount of discount on t-shirts at each price.

c. Write a number sentence that describes how much discount you will receive on any T-shirt that is offered at a 30% discount.

B

cost	$
17	5.10
18	5.40
19	5.70
20	6.00
50	15.00

C 30% of cost = discount

- Start w/ numerical task and vary it by changing one of the quantities
- Get students to move from specific to general, more algebraic

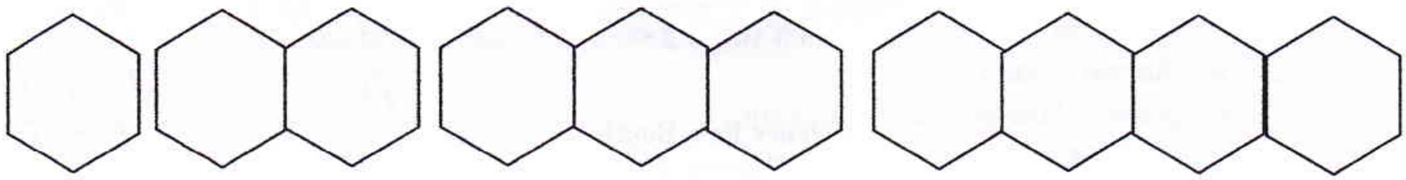
ALGEBRAFYING: Pattern building, conjecturing, generalizing, justifying

KEY

Hexagon Pattern Train Task

Trains 1, 2, 3, and 4 (shown below) are the first 4 trains in the hexagon pattern. The first train in this pattern consists of one regular hexagon. For each subsequent train, one additional hexagon is added.

6 10 14 18



train 1 train 2 train 3 train 4

# HEX	P
1	6
2	10
3	14
4	18
5	22

$P = 4(x-1)+6$

$P = 4x-4+6$

$P = 4x+2$

OR

$U_1 = 6$

$U_{n+1} = U_n + 4$

$U_n = U_{n-1} + 4$

1. Compute the perimeter for each of the first four trains.

2. Draw the fifth train and compute the perimeter of the train.

$P = 22$

3. Make some observations that could help you describe the perimeter of larger trains.

\# hexagons increases by one
\# sides increases by 4

4. Determine the perimeter of the 25th train without constructing it. 102

5. Write a description that could be used to compute the perimeter of any train in the pattern. Explain how you know it will always work.

6. Write an equation that could be used to compute the perimeter of any train in the pattern.

$P = 4x+2$

7. What is the real world meaning of rate of change?

Extension: How could you find the perimeter of a train that consisted of triangles? Squares? Pentagons? Can you write a general description that could be used to find the perimeter of a train of any regular polygons?

The Hexagon Train Task has been adapted from *Visual Mathematics Course 1, Lesson 16-30* published by The Math Learning Center, Salem, Oregon, 1995.

△'s

n	P
1	3
2	4
3	5
n	$1(n-1)+3$
	$n+2$

□'s

n	P
1	4
2	6
3	8
n	$2(n-1)+4$
	$2n+2$

⬠'s

n	P
1	5
2	8
3	11
n	$3(n-1)+5$
	$3n+2$

$A=49$ $\quad P=4s$

$A=s^2$ $\quad P=4\cdot 7$

$49=s^2$ $\quad P=28$

$7=s$ $\quad$ 28 ft of rope

Science Fair Booth
Version 1

Jeremiah and Haley use a piece of rope to mark a square space for their booth at the science fair. The area of their space is 49 square feet. What is the length of the rope that Jeremiah and Haley need to use?

Science Fair Booth
Version 2

Jeremiah and Haley use a piece of rope to mark a rectangular space for their booth at the science fair. They have a rope that is 28 feet long.

a. What is the area of largest rectangular space they can enclose? Explain how you know.

b. What if the rope was 36 feet long? 48 feet? What would be the area of largest rectangular space they could enclose? Explain how you know.

c. What are the dimensions of the largest rectangular space that can be enclosed with *any* amount of fence? How did you come to your conclusion?

A. $P=28$

$P=2(l+w)$

$28=2(l+w)$

$14=l+w$

l	w	A
1	13	13
2	12	24
3	11	33
4	10	40
5	9	45
6	8	48
7	7	49
8	6	48

B. 36 ft rope $\quad s=9 \rightarrow A=s^2$

$A=81\ ft^2$

48 ft rope $\quad s=12 \quad A=s^2$

$A=144\ ft^2$

C. -Make table

-Find when length = width (get square)

-Divide perimeter by 4 to find length of side of square

EXTEND: BOOTH HAS 3 SIDES. FIND LARGEST AREA

Characteristics of mathematical tasks at four levels of Cognitive demand.

Lower-Level Demands	**Higher-Level Demands**
Memorization • involve either reproducing previously learned facts, rules, formulae or definitions OR committing facts, rules, formulae or definitions to memory • cannot be solved using procedures because a procedure does not exist or because the time frame in which the task is being completed is too short to use a procedure • are not ambiguous. Such tasks involve exact reproduction of previously-seen material and what is to be reproduced is clearly and directly stated • have no connection to the concepts or meaning that underlie the facts, rules, formulae or definitions being learned or reproduced	Procedures With Connections • focus students' attention on the use of procedures for the purpose of developing deeper levels of understanding of mathematical concepts and ideas • suggest pathways to follow (explicitly or implicitly) that are broad general procedures that have close connections to underlying conceptual ideas as opposed to narrow algorithms that are opaque with respect to underlying concepts • usually are represented in multiple ways (e g , visual diagrams, manipulatives, symbols, problem situations). Making connections among multiple representations helps to develop meaning • require some degree of cognitive effort. Although general procedures may be followed, they cannot be followed mindlessly Students need to engage with the conceptual ideas that underlie the procedures in order to successfully complete the task and develop understanding.
Procedures Without Connections • are algorithmic. Use of the procedure is either specifically called for or its use is evident based on prior instruction, experience, or placement of the task • require limited cognitive demand for successful completion. There is little ambiguity about what needs to be done and how to do it • have no connection to the concepts or meaning that underlie the procedure being used • are focused on producing correct answers rather than developing mathematical understanding • require no explanations or explanations that focuses solely on describing the procedure that was used	Doing Mathematics • require complex and non-algorithmic thinking (i e., there is not a predictable, well-rehearsed approach or pathway explicitly suggested by the task, task instructions, or a worked-out example) • require students to explore and understand the nature of mathematical concepts, processes, or relationships • demand self-monitoring or self-regulation of one's own cognitive processes • require students to access relevant knowledge and experiences and make appropriate use of them in working through the task • require students to analyze the task and actively examine task constraints that may limit possible solution strategies and solutions • require considerable cognitive effort and may involve some level of anxiety for the student due to the unpredictable nature of the solution process required

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Stein, Smith, Henningsen, & Silver, 2000, p 16