Welcome!

- **Fine-Tuning Mathematics Curriculum and Instruction**
  Grades Pre K-8
  The presentation will include specific examples of how a narrowing of the curriculum (to less than “a mile wide”) might be accompanied by an increase in the depth of learning (to more than “an inch deep”). The eight mathematical practices will be highlighted as participants “do” mathematics aligned with the Common Core.

AMTNJ
2012

COMMON CORE
STATE STANDARDS IN MATHEMATICS
PREPARING AMERICA'S STUDENTS FOR COLLEGE & CAREER

Fine-Tuning Mathematics Curriculum and Instruction to Implement the Common Core State Standards in MATHEMATICS

Grades Pre K-8

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Common Core Implementation

Core Curriculum Content Standards	Adoption by State Board	Implementation by Districts
Common Core State Standards for Mathematics	June 16, 2010	K, 1, 2 Sept. 2011 3, 4, 5 & H.S. Sept. 2012 6, 7, 8 Sept. 2013

The NJ ASK 6, 7, & 8 Mathematics
and
the NJ ASK 3, 4, & 5 Mathematics
will all be aligned with the Common Core State Standards
for the April and May 2013 administrations.

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2012

“A mile wide and an inch deep”

Approximately 15 years ago, the phrase “a mile wide and an inch deep” entered the discourse on mathematics education in the United States. Growing out of the results from the Third International Mathematics and Science Study (TIMSS), the phrase was a criticism of expansive, unfocused, and purportedly less rigorous mathematics curricula in the United States as compared to curricula in other nations. (Schmidt, McKnight, & Raizen, 1997).

“A mile wide and an inch deep”

No clear focus!

Spread too thin!

- Number of topics
- Thickness of textbooks

What the United States Can Learn from Singapore's World-Class Mathematics System: An Exploratory Study

Alan Ginsburg
United States Department of Education
Policy and Program Studies Service (PPSS)
Steven Leinwand
American Institutes for Research
Terry Anstrom
American Institutes for Research
Elizabeth Pollock
American Institutes for Research

AIR

January 28, 2005

What the United States Can Learn from Singapore's World-Class Mathematics System: An Exploratory Study

"Analysis of these evidentiary streams finds Singaporean students more successful in mathematics than their U.S. counterparts because *Singapore has a world-class mathematics system with quality components aligned to produce students who learn mathematics to mastery.* These components include Singapore's highly logical national mathematics framework, mathematically rich problem-based textbooks, challenging mathematics assessments, and highly qualified mathematics teachers whose pedagogy centers on teaching to mastery. Singapore also provides its mathematically slower students with an alternative framework and special assistance from an expert teacher."

AIR

January 28, 2005

What the United States Can Learn from Singapore's World-Class Mathematics System: An Exploratory Study

"*The U.S. mathematics system does not have similar features*. It lacks a centrally identified core of mathematical content that provides a focus for the rest of the system. Its traditional textbooks emphasize definitions and formulas, not mathematical understanding; its assessments are not especially challenging; and too many U.S. teachers lack sound mathematics preparation. At-risk students often receive special assistance from a teacher's aide who lacks a college degree. As a result, the United States produces students who have learned only to mechanically apply mathematical procedures to solve routine problems and who are, therefore, not mathematically competitive with students in most other industrialized countries.."

AIR

January 28, 2005

What the United States Can Learn from Singapore's World-Class Mathematics System: An Exploratory Study

"…reduce breadth of coverage and deepen topic instruction." (p. xii)

AIR

January 28, 2005

One intent of the Common Core State Standards in Mathematics is to provide

Kindergarten

- **Two critical areas of focus:**
  - Representing, relating, and operating on whole numbers, initially with sets of objects
  - Describing shapes and space

Kindergarten: What's Different

- No time
- No calendar
- No graphing
- No money
- No non-standard measurement
- No fractions
- No patterning
- No estimation

First Grade

- **Four critical areas:**
  - Developing understanding of addition and subtraction, and strategies for addition and subtraction within 20
  - Whole number relationships and place value
  - Linear measurement and measuring lengths
  - Composing and decomposing geometric shapes

Second Grade

- **Four critical areas:**
  - Extending understanding of base-ten notations
  - Building fluency with addition and subtraction
  - Using standard units of measure
  - Describing and analyzing shapes

Fifth Grade

- **Three critical areas:**
  - Developing fluency with addition and subtraction of fractions, and developing understanding of multiplication and division of fractions
  - Extending division to 2-digit divisors, integrating decimal fractions into the place value system and developing understanding of operations with decimals to hundredths, and developing fluency with whole number and decimal operations
  - Developing understanding of volume

Eighth Grade

- **Three critical areas:**
  - Formulating and reasoning about expressions and equations, including modeling data with a linear equation, and solving linear equations and systems of linear equations
  - Grasping the concept of a function and using functions to describe quantitative relationships
  - Analyzing two- and three-dimensional space and figures using distance, angle, similarity, and congruence, and understanding and applying the Pythagorean Theorem

What the United States Can Learn from Singapore's World-Class Mathematics System: An Exploratory Study

"…reduce breadth of coverage and deepen topic instruction." (p. xii)

AIR

January 28, 2005

Standards for Mathematical Practice

1 Make sense of problems and persevere in solving them.

2 Reason abstractly and quantitatively.

3 Construct viable arguments and critique the reasoning of others.

4 Model with mathematics.

5 Use appropriate tools strategically.

6 Attend to precision.

7 Look for and make use of structure.

8 Look for and express regularity in repeated reasoning.

The Challenge of "Deeper" Learning

The emphasis in many classrooms has been on getting the answer to a problem or series of problems.

Phil Daro

AMTNJ 2012

"…reduce breadth of coverage and deepen topic instruction."

The Challenge of "Deeper" Learning

- The challenge is to have students learn mathematics from the problems, and in many cases that means having them keep their brains turned on even after the answer has been found.
  - Diverting students' attention from mechanistically following a memorized algorithm.
  - Diverting students' attention from just finding the answer.

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"…reduce breadth of coverage and deepen topic instruction."

Don't provide a question …

Provide a situation to discuss!

Grade 8

Train A leaves the station going 50 miles per hour and continues at that speed.

Train B leaves the station 3 hours later going 60 mph.

- How long will it take train B to overtake train A?
- When train B departs, how far behind train A is it?
- How far will they both have gone when train B catches up to train A?

[*Students identify possible questions that arise from the context.*]

--Phil Daro,
April 20, 2012

Don't provide a question …

Provide a situation to discuss!

Mathematical Practice:

Grade 8

1. Make sense of problems and persevere in solving them.

4. Model with mathematics.

CCSS 8.EE.8 Analyze and solve pairs of simultaneous linear equations.

a. Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously.

b. Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection. *For example, $3x + 2y = 5$ and $3x + 2y = 6$ have no solution because $3x + 2y$ cannot simultaneously be 5 and 6.*

c. Solve real-world and mathematical problems leading to two linear equations in two variables. *For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.*

Don't provide a question …

Provide a situation to discuss!

Kindergarten

Juan and Naomi each reach into a cookie jar.

Juan takes out 5 cookies.

Naomi takes our 3 cookies.

[*Students identify possible questions that arise from the context.*]

Don't provide a question …

Provide a situation to discuss!

Mathematical Practice:

1. Make sense of problems and persevere in solving them.

Kindergarten

CCSS K.CC.6 Identify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group, e.g., by using matching and counting strategies. [*Include groups with up to ten objects.]*

CCSS K.OA.2 Solve addition and subtraction word problems, and add and subtract within 10, e.g., by using objects or drawings to represent the problem.

Provide a question, and ask …

Why is it the answer?

Grade 1

Example for comparing indirectly:

Two students each make a dough "snake." Given a tower of cubes, each student compares his/her snake to the tower. Then students make statements such as, "My snake is longer than the cube tower and your snake is shorter than the cube tower. So, my snake is longer than your snake."

--Arizona Mathematics Standards, updated May 20, 2011

Provide a question, and ask …

Why is it the answer?

Mathematical Practice:

3. Construct viable arguments and critique the reasoning of others.

Grade 1

- **CCSS 1.MD.1** Order three objects by length; compare the lengths of two objects indirectly by using a third object.

Give them the answer, and ask …

Why is it the answer?

Grade 5

4/5 is closer to one than is 5/4.

Explain why using the number line.

--Phil Daro,
April 20, 2012

Give them the answer, and ask …

Why is it the answer?

Mathematical Practice:

3. Construct viable arguments and critique the reasoning of others.

Grade 5

CCSS 5.NF.2 Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. *For example, recognize an incorrect result* $2/5 + 1/2 = 3/7$, *by observing that* $3/7 < 1/2$.

Give them the answer, and seek …

Alternative solution strategies

Grade 6

CCSS 6.G.1 Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.

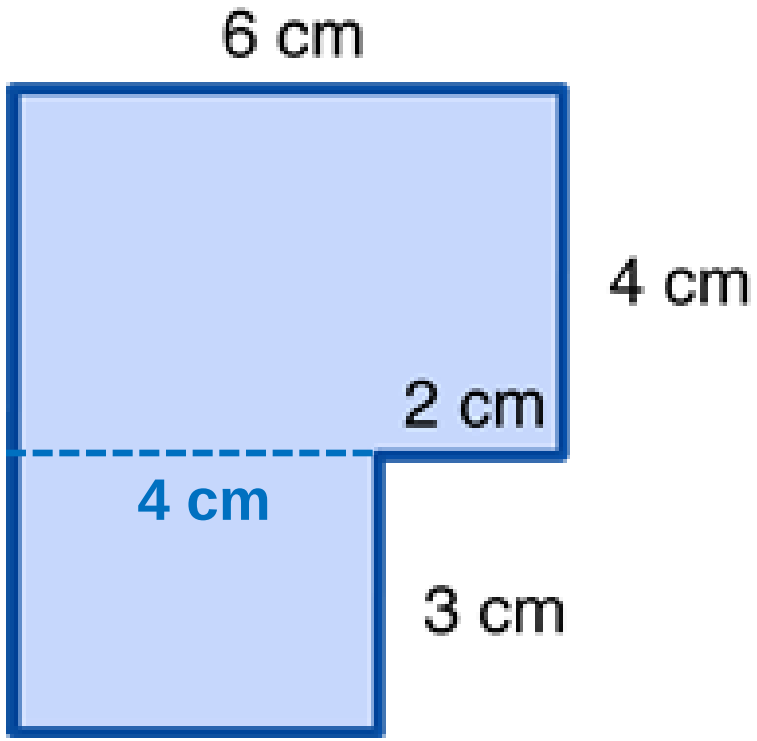
$36\ cm^2$

Give them the answer, and seek …

Alternative solution strategies

Grade 6

- [CCSS 6.G.1] Find the area of this room:

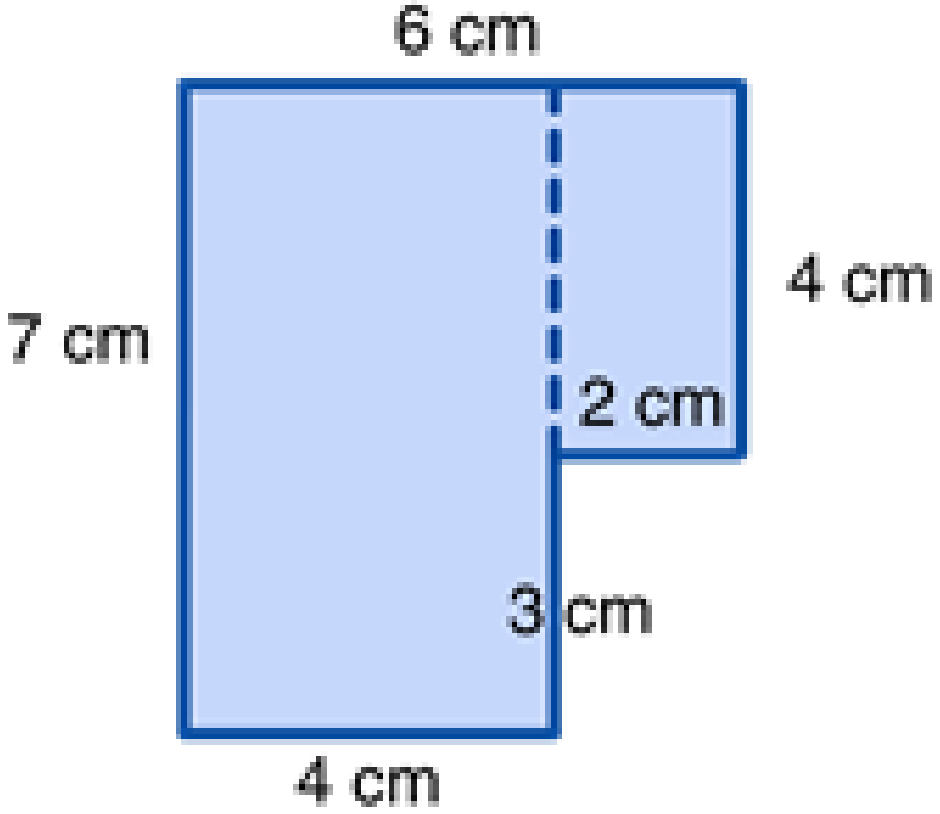


Give them the answer, and seek …

Alternative solution strategies

Grade 6

- [CCSS 6.G.1] Find the area of this room:



Give them the answer, and seek …

Alternative solution strategies

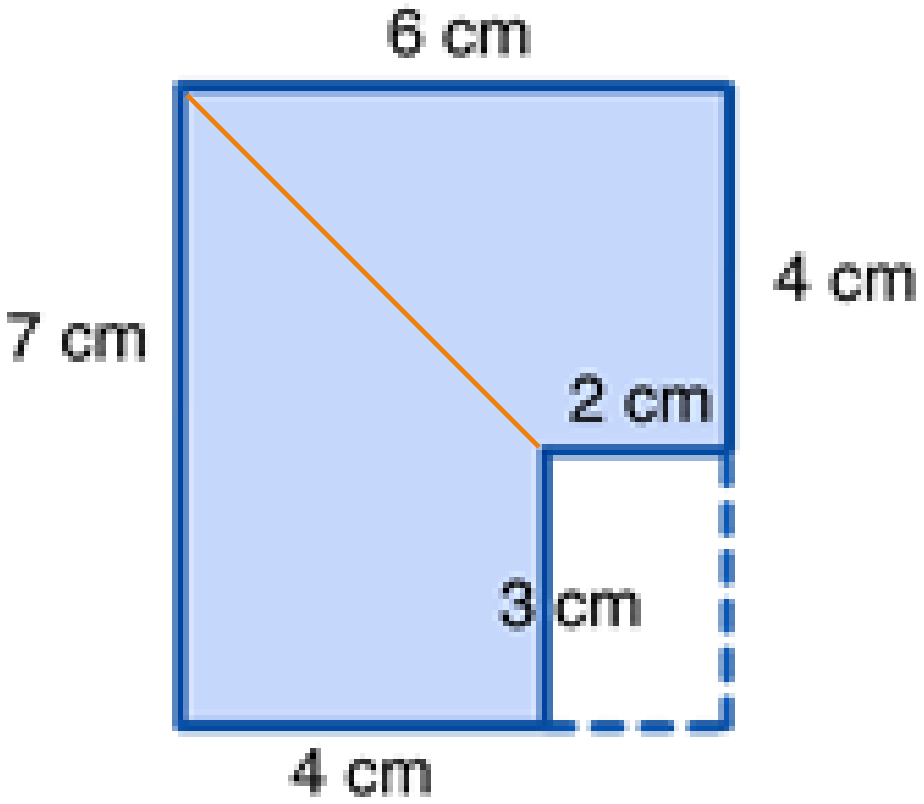
Mathematical Practice:

4. Model with mathematics.

7. Look for and make use of structure.

Grade 6

- [CCSS 6.G.1] Find the area of this room:



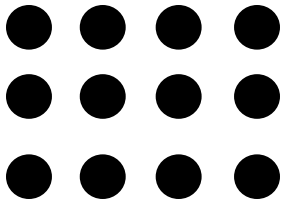
Notes:
Doing mathematics
Special Education

Give them the answer, and seek … .

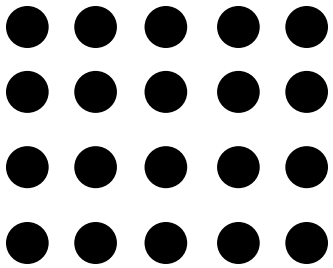
Alternative solution strategies

Grade 2

- **[CCSS 2.OA.4]** Students may arrange any set of objects into a rectangular array. Objects can be cubes, buttons, counters, etc. Objects do not have to be square to make an array. Students then write equations that represent the total as the sum of equal addends as shown below.



4 + 4 + 4 = 12



5 + 5 + 5 + 5 = 20

--Arizona Mathematics Standards, updated May 20, 2011

Give them the answer, and seek … .

Alternative solution strategies

Mathematical Practice:

4. Model with mathematics.

7. Look for and make use of structure.

Grade 2

CCSS 2.OA.4 Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends.

3. Construct viable arguments and critique the reasoning of others.

Don't give them the answer, and seek …

Alternative solution strategies

Grade 8

- **[CCSS 8.EE.5]** The graph summarizes information about the trips that 4 students took over the spring break. According to the graph, which student had the greatest average speed?

  (A) Carmen

  (B) George

  (C) Ali

  (D) Jo

--Elizabeth Marquez & Broderick Boxley (2003)
Pathwise Instructional Tools, ETS

Don't give them the answer, and seek …

Alternative solution strategies

Mathematical Practice:

2. Reason abstractly and quantitatively.

4. Model with mathematics.

Grade 8

- **CCSS 8.EE.5** Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. *For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.*

Give them an open-ended question with …

Multiple Answers

Grade 7

1. Discuss with your partner how you might break the strand of linguini into pieces that cannot be formed into a triangle by connecting the pieces at their endpoints.
2. Break the strand of linguini into pieces that you believe will not form a triangle.
3. Try to form a triangle with the pieces by connecting them at their endpoints. (If your strategy was correct, you should not be able to form the triangle.)
4. Compare your solution with the solution obtained by another group. If neither set of pieces can be formed into a triangle, is it for the same reason?

Give them an open-ended question with …

Multiple Answers

Mathematical Practice:

6. Attend to precision.

8. Look for and express regularity in repeated reasoning.

Grade 7

- **CCSS 7.G.2** Draw (freehand, with ruler and protractor, and with technology) geometric shapes with given conditions. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle.

3. Construct viable arguments and critique the reasoning of others.

Give them an open-ended question with …

Multiple Answers

Grade 3

- **CCSS 3.MD.2** Students identify 5 things that weigh about one gram. They record their findings with words and pictures. (Students can repeat this for 5 grams and 10 grams.)
- This activity helps develop gram benchmarks. One large paperclip weighs about one gram. A box of large paperclips (100 clips) weighs about 100 grams so 10 boxes would weigh one kilogram..

--Arizona Mathematics Standards, updated May 20, 2011

Give them an open-ended question with …

Multiple Answers

Mathematical Practice:

2. Reason abstractly and quantitatively.

6. Attend to precision.

Grade 3

- **CCSS 3.MD.2** Measure and estimate liquid volumes and masses of objects using standard units of grams (g), kilograms (kg), and liters (l). Add, subtract, multiply, or divide to solve one-step word problems involving masses or volumes that are given in the same units, e.g., by using drawings (such as a beaker with a measurement scale) to represent the problem.

Give them the answer, and ask them to …

Interpret the answer within the context!

--Phil Daro

Grade 6

- **CCSS 6.RP.2** Diane ran 30 laps in 15 minutes.

  30 15 = 2

  What does the 2 tell you about Diane's run?

  laps/minute

  15 30 = 0.5

  What does that tell you?

  minutes/lap

  How do you know that's what it tells you?

Give them the answer, and ask them to …

Interpret the answer within the context!

Grade 6

- **CCSS 6.RP.2** Diane ran 30 laps in 15 minutes.

  30 ÷ 15 = 2

  What does the 2 tell you about Diane's run?

  laps/minute

  15 ÷ 30 = 0.5

  What does that tell you?

  minutes/lap

  How do you know that's what it tells you?

$\frac{\text{15 minutes}}{\text{30 laps}}$

0.5 minutes/lap

Give them the answer, and ask them to …

Interpret the answer within the context!

Mathematical Practice:

1. Make sense of problems and persevere in solving them.

2. Reason abstractly and quantitatively.

Grade 6

- **CCSS 6.RP.2** Understand the concept of a unit rate a/b associated with a ratio $a:b$ with $b \neq 0$, and use rate language in the context of a ratio relationship. *For example, "This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is 3/4 cup of flour for each cup of sugar." "We paid $75 for 15 hamburgers, which is a rate of $5 per hamburger."*

6. Attend to precision.

7. Look for and make use of structure.

Give them the answer, and ask them to …

Interpret the answer within the context!

Grade 4

- **CCSS 4.OA.3** Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.
- **CCSS 4.MD.2** Use the four operations to solve word problems involving
  - distances,
  - intervals of time,
  - liquid volumes,
  - masses of objects, and
  - money,

  including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale.

Give them the answer, and ask them to …

Interpret the answer within the context!

Grade 4

- **CCSS 4.OA.3 & 4.MD.2**

  Describe a situation in which you might want to divide 12 by 3 **(12 ÷ 3)**.

12 feet of ribbon divided equally among 3 students

12 photographs shared equally among 3 students

12 crayons divided into packages of 3 crayons each

Give them the answer, and ask them to …

Interpret the answer within the context!

Mathematical Practice:

1. Make sense of problems and persevere in solving them.

4. Model with mathematics.

Grade 4

- **CCSS 4.OA.3 & 4.MD.2**

  Describe a situation in which you might want to divide 12 by 3 **(12 ÷ 3)**.

2 feet of ribbon divided equally among 3 students

24 inches ÷ 3 students

2 photographs shared equally among 3 students

Give students problems that can't be done

Welcome to Mathtown

Founded	1890
Population	3000
Altitude (ft)	1000
Total:	5890

--Marilyn Burns

Thank you!

Questions?

Concerns?

"…reduce breadth of coverage and deepen topic instruction."

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