

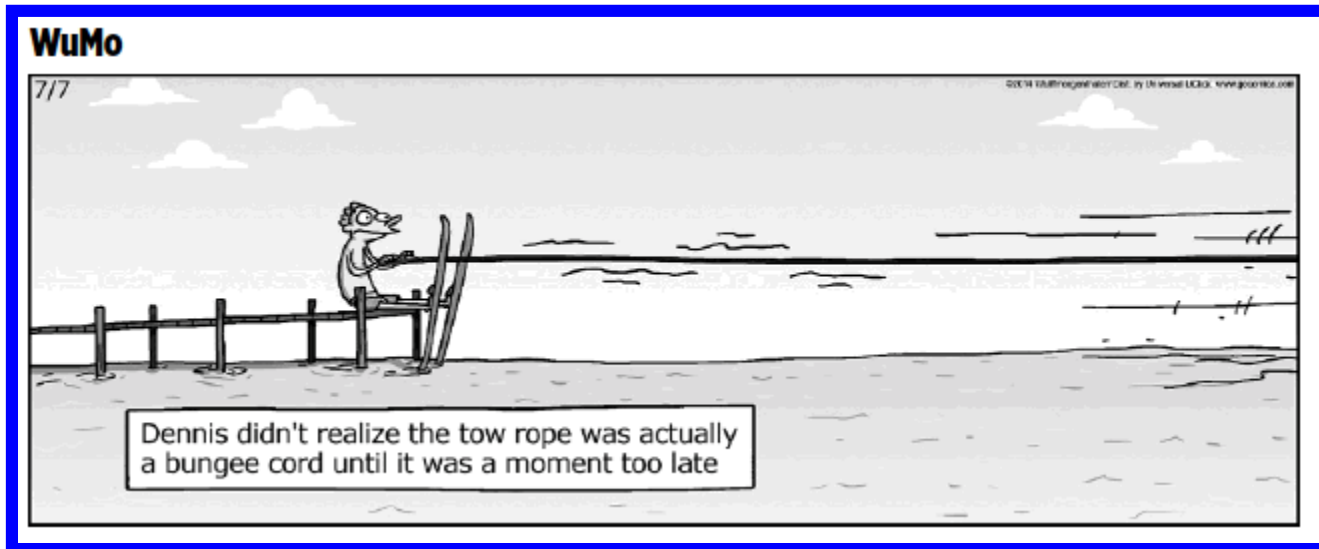
Don't Be So Symbol Minded: *Incorporating Algebraic Thinking Across the Grades*

Jim Rubillo

JRubillo@verizon.net

When should students experience the concepts of algebra? This is a “hot topic” across the country. This session will examine ways that schools are revising and articulating their curriculum to ensure that all students are better prepared whenever they first enter their first formal algebra course. Practical examples and approaches will be presented.

School Day One for a Teacher



Jim didn't realize the tow rope was actually a bungee cord until it was a moment too late

What You Are Up Against!



5 FOR \$3
Piece

3 FOR \$1.99
5 FOR \$3.00
10 FOR \$8.99

Heat up
your taste buds

A digital sign with a dark background and yellow and white text. It features a large '5 FOR \$3' offer and lists other bundle prices. At the bottom, it says 'Heat up your taste buds' with an image of chicken nuggets.

get
50% off
or half price,
whichever is less.

A sign with a colorful, pixelated background and a dashed border. It offers a 50% discount, stating 'or half price, whichever is less.'

MSRP PRICE \$899⁰⁰

INSTANT SAVINGS: \$400⁰⁰

PRICE AFTER SAVINGS \$628⁵²

A green price tag with black text. It shows the MSRP, a \$400 instant savings, and the final price after savings.

\$24⁹⁹ sale
09/07 - 09/13

Metaphor carson
\$19⁹⁸ reg.
5.5-10, 11

look for: 42067

054/42067 P ITM 07/13
BAS DO POG: 54030 Seq.

A price tag in a clear plastic holder. It features a red 'sale' sticker with a price of \$24.99 and a date range. Below it, the product name 'Metaphor carson' is listed with a regular price of \$19.98 and a date range of 5.5-10, 11. A 'look for' number and a barcode are also present.

Correct, But...



And Some Need Detailed Help...

CLEARANCE

90% OFF
our already low prices!
Remaining Christmas Merchandise

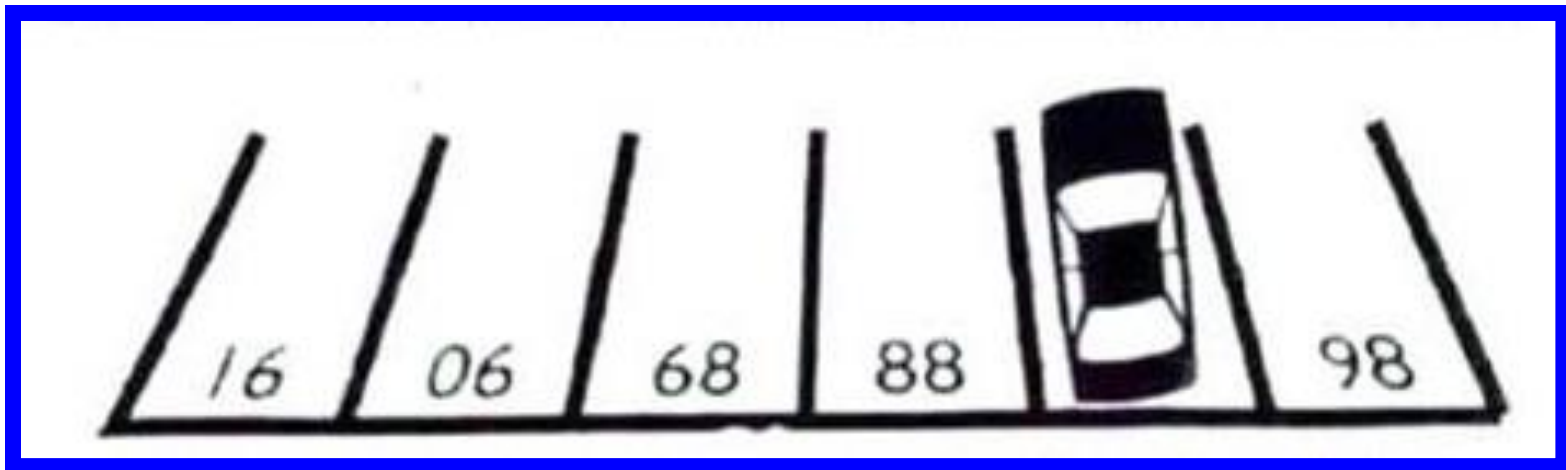
Prices reduced at register.

Was	NOW	Was	NOW	Was	NOW
\$1.00 \$0.10	\$7.50 \$0.75	\$25.00	... \$2.50
\$1.50 \$0.15	\$8.00 \$0.80	\$28.00	... \$2.80
\$1.80 \$0.18	\$9.00 \$0.90	\$29.99	... \$2.99
\$2.00 \$0.20	\$10.00	... \$1.00	\$30.00	... \$3.00
\$2.20 \$0.22	\$12.00	... \$1.20	\$30.50	... \$3.05
\$2.50 \$0.25	\$12.50	... \$1.25	\$32.00	... \$3.20
\$3.00 \$0.30	\$13.50	... \$1.35	\$35.00	... \$3.50
\$3.50 \$0.35	\$14.00	... \$1.40	\$36.00	... \$3.60
\$4.00 \$0.40	\$15.00	... \$1.50	\$39.99	... \$3.99
\$4.50 \$0.45	\$16.00	... \$1.60	\$40.00	... \$4.00
\$4.99 \$0.49	\$16.50	... \$1.65	\$47.50	... \$4.75
\$5.00 \$0.50	\$17.00	... \$1.70	\$49.99	... \$4.99
\$5.30 \$0.53	\$18.00	... \$1.80	\$60.00	... \$6.00
\$6.00 \$0.60	\$20.00	... \$2.00	\$69.99	... \$6.99
\$6.50 \$0.65	\$24.00	... \$2.40	\$79.99	... \$7.99
\$7.00 \$0.70	\$24.99	... \$2.49		

And Some Need Help with Punctuation...



An Assessment Question: What Grade?



What is the Parking Spot # where the car is parked?

What is Algebra?

**The intensive study of
the last three letters of
the alphabet.**

Algebra is a K-12 Strand

All students should have access to algebra in a pre-K–12 mathematics curriculum, including opportunities to generalize, model, and analyze situations that are purely mathematical and ones that arise in real-world phenomena. Algebraic ideas need to evolve across grades as a way of thinking and valuing structure with integrated sets of concepts, procedures, and applications.

NCTM Position Statement : **Algebra as a Strand of School Mathematics for All Students (April 2014)**

Algebra is **not** confined to a course or set of courses in the school curriculum; rather, it is a strand that unfolds across a pre-K–12!

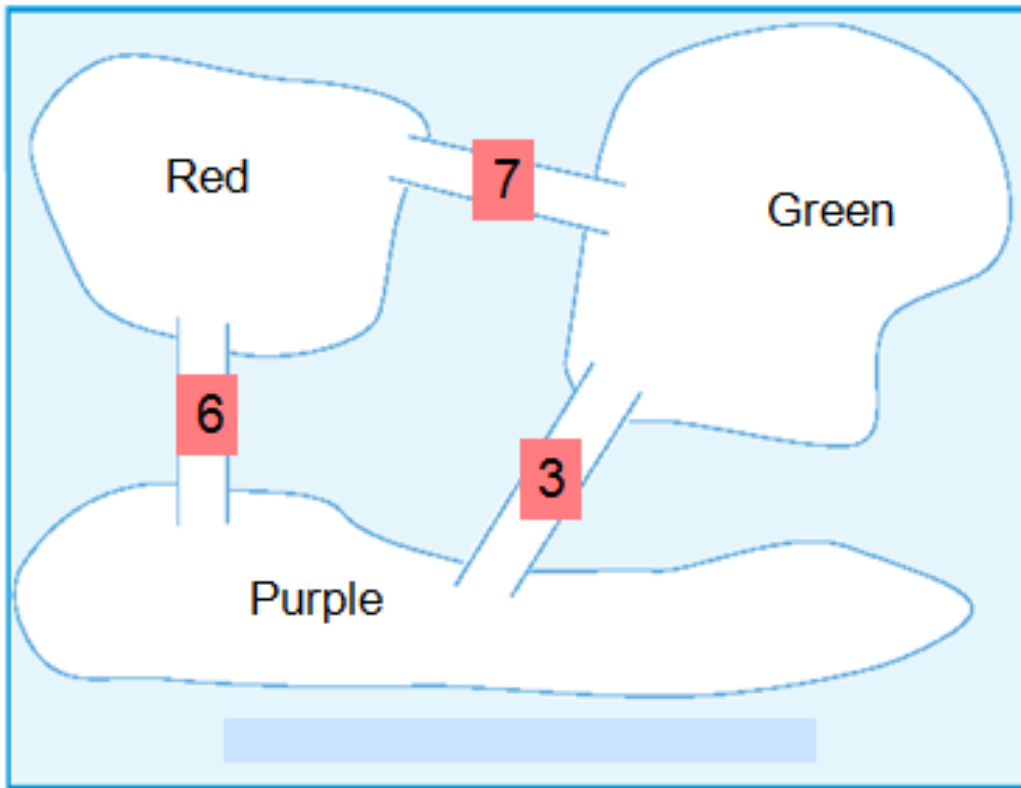
The Algebra Strand should contain:

- Exploring and extending patterns
- Representing mathematical ideas with symbols and objects
- Using mathematical models to represent quantitative relationships
- Analyzing change in various contexts

SHOW ME!

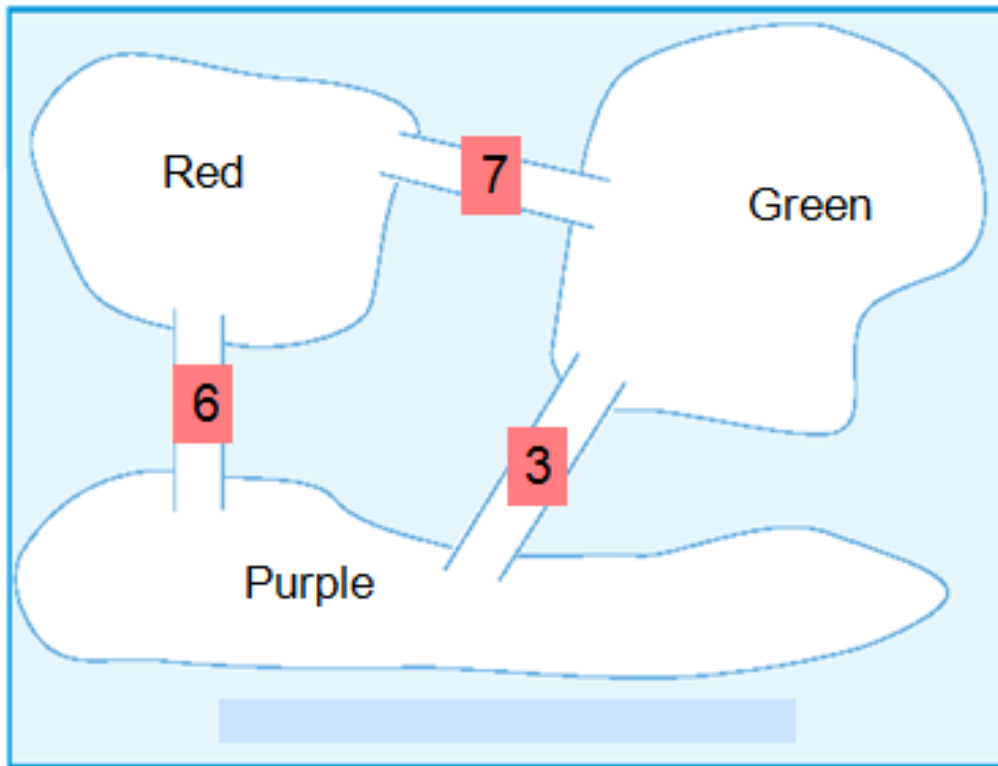
Kindergarten or First Grade

How many cows in each pasture?



Three Unknown Placeholders

How many cows in each pasture?



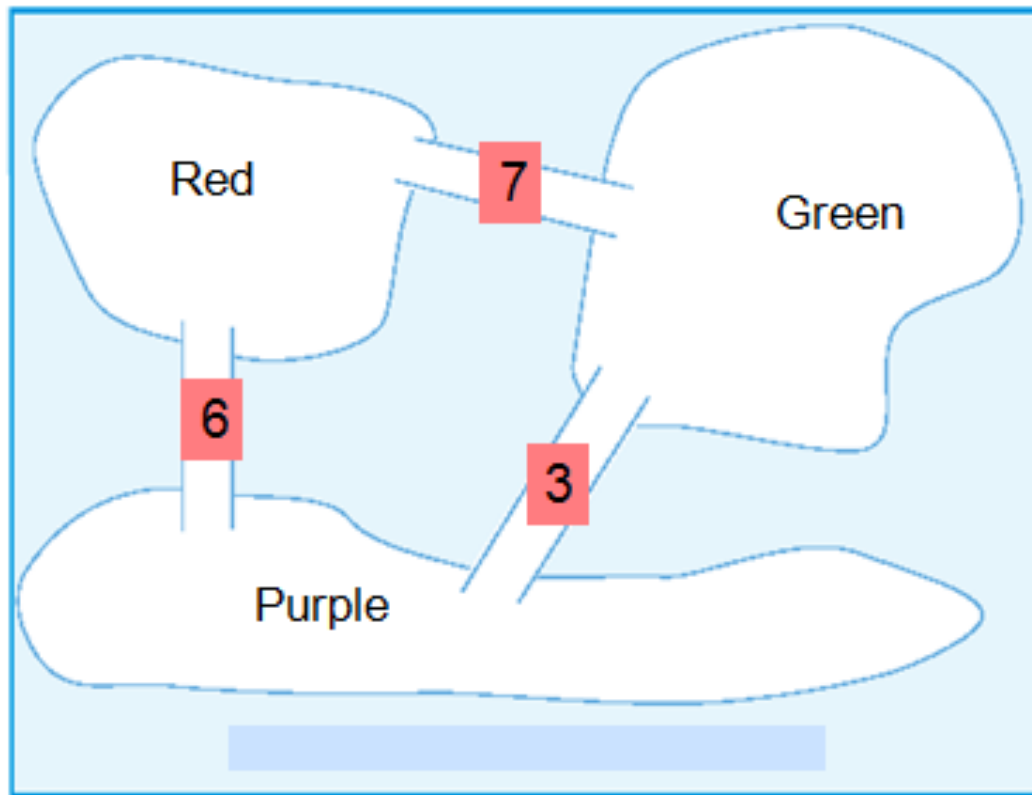
$$\text{Red} + \text{Green} = 7$$

$$\text{Red} + \text{Purple} = 6$$

$$\text{Purple} + \text{Green} = 3$$

Three Unknown Placeholders

How many cows in each pasture?

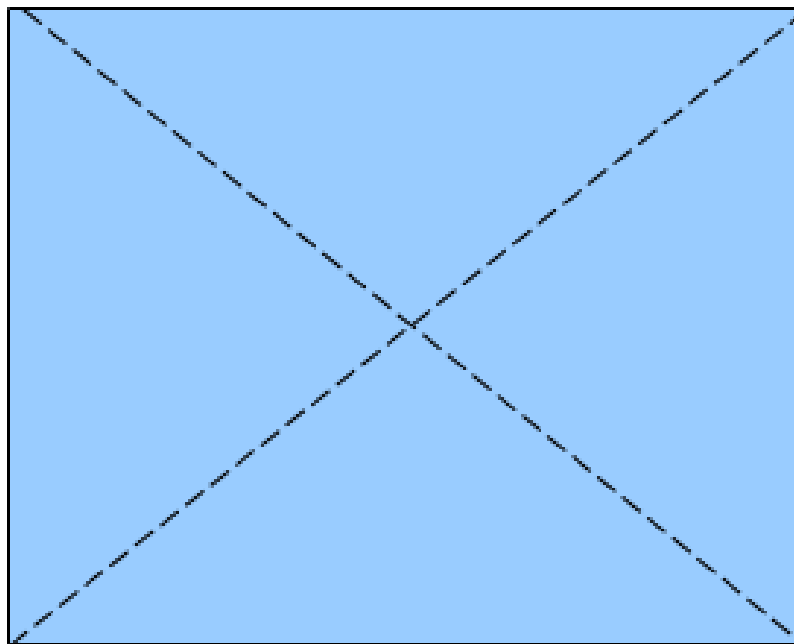


Red = 5

Purple = 1

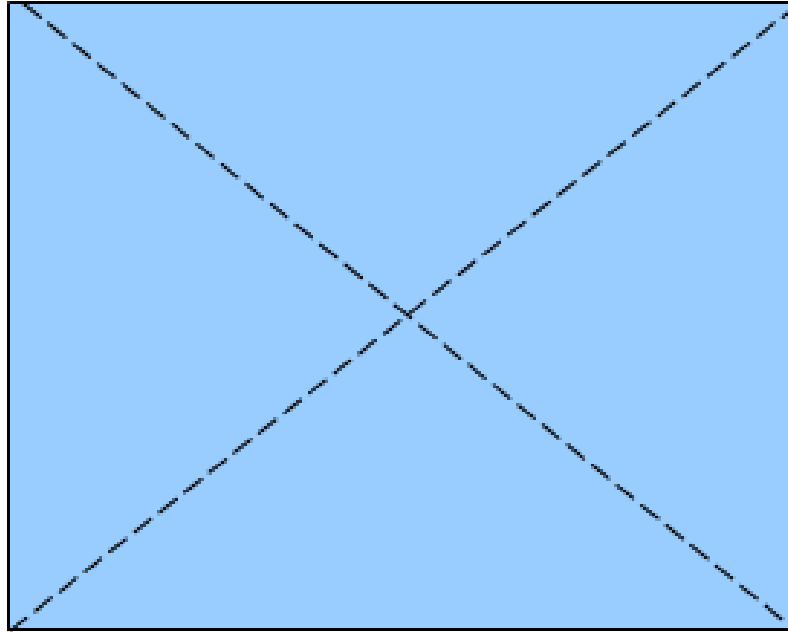
Green = 2

Follow the Fold(s)



Folds	0	1	2	3	4	5	6	7	8	9	10
Sides	4	5	6	5	4	5	6	5	4	5	6

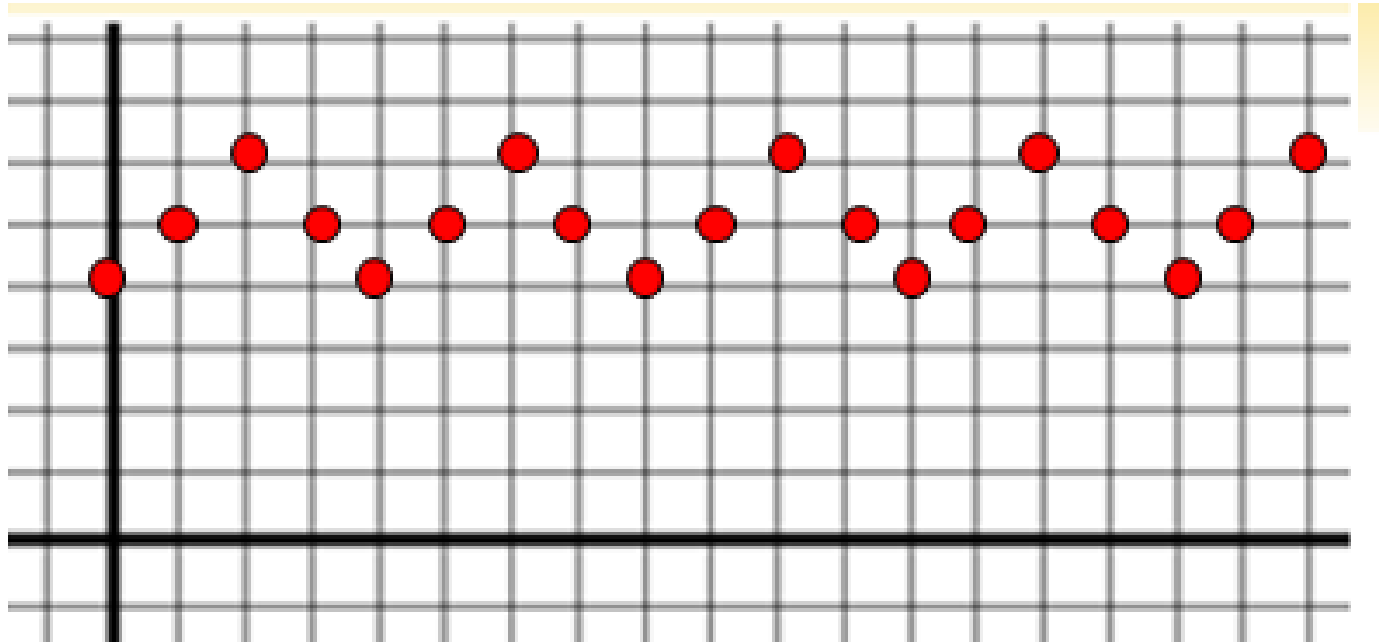
Follow the Fold(s)



Folds	44	--	49	--	67	--	82	--	876	--
Sides	4		5		5		6		4	

What's the Graph?

Sides



Folds

Folds	0	1	2	3	4	5	6	7	8	9	10
Sides	4	5	6	5	4	5	6	5	4	5	6

What's My Rule?

Number of Folds (N)	Number of Sides
Remainder is 0 when N is divided by 4	4
Remainder is 1 when N is divided by 4	5
Remainder is 2 when N is divided by 4	6
Remainder is 3 when N is divided by 4	5

Playing with the Four Basic Operations

$$1 + 1 = 2$$

$$1 - 1 = 0$$

$$1 \times 1 = 1$$

$$1 \div 1 = 1$$

$$\text{Total} = 4$$

Playing with the Four Basic Operations

$$2 + 2 = 4$$

$$2 - 2 = 0$$

$$2 \times 2 = 4$$

$$2 \div 2 = 1$$

$$\text{Total} = 9$$

Playing with the Four Basic Operations

$$3 + 3 = 6$$

$$3 - 3 = 0$$

$$3 \times 3 = 9$$

$$3 \div 3 = 1$$

$$\text{Total} = 16$$

Playing with the Four Basic Operations

$$4 + 4 = 8$$

$$4 - 4 = 0$$

$$4 \times 4 = 16$$

$$4 \div 4 = 1$$

$$\text{Total} = 25$$

Playing with the Four Basic Operations

Willing to Predict?

<i>n</i>	1	2	3	4	5	6	7	8	9
<i>Total</i>	4	9	16	25	36	49	64	81	100

Playing with the Four Basic Operations

Willing to Generalize?

<i>n</i>	1	2	3	4	5	6	7	..	<i>n</i>
<i>Total</i>	4	9	16	25	36	49	64		

$$***n + n = 2n***$$

$$***n - n = 0***$$

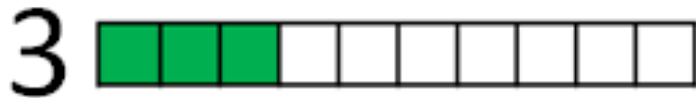
$$***n \times n = n^2***$$

$$***n \div n = 1 \quad \text{when } n \neq 0***$$

$$**Total = *n^2 + 2n + 1 = (n + 1)^2***$$

**But What
About the
BASICS Jim?**

Smart Flash Cards - Addition



?



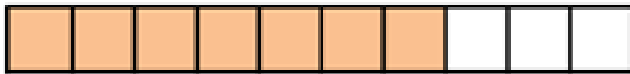
?

Smart Flash Cards - Addition

$$2 + 4 =$$



$$7 + 5 =$$

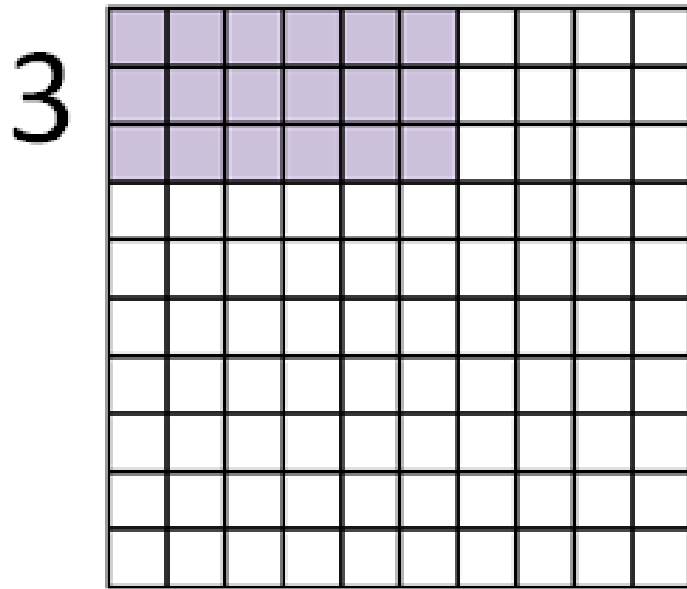


$$9 + 7 =$$

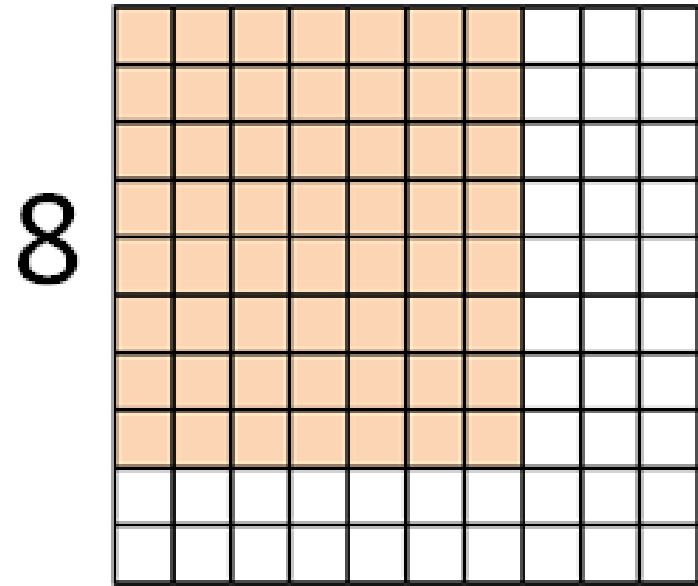


Smart Flash Cards - Multiplication

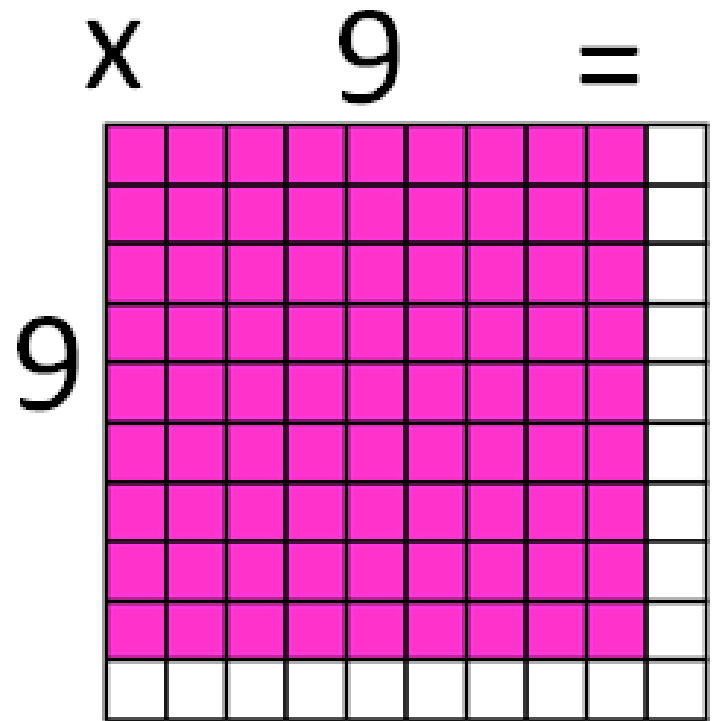
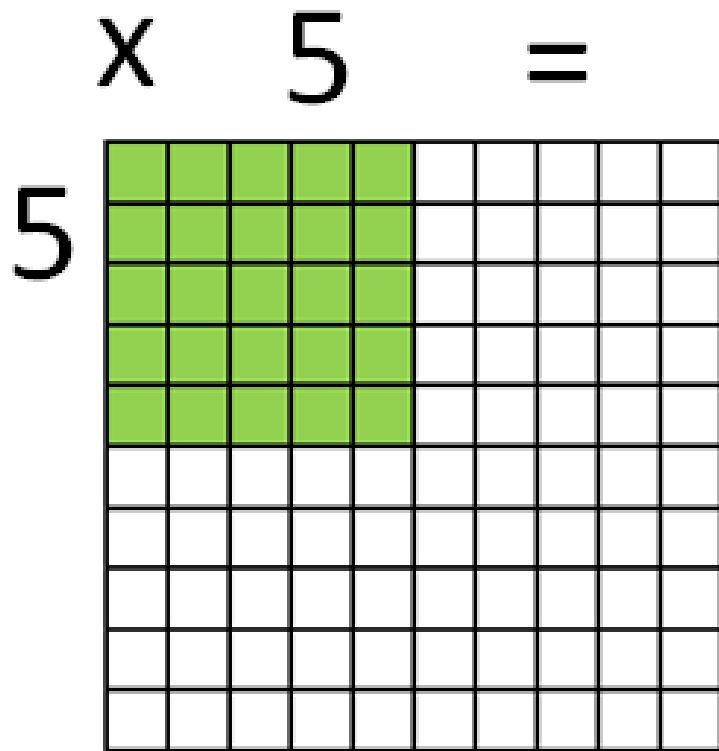
$$x \quad 6 \quad =$$



$$x \quad 7 \quad =$$



Smart Flash Cards - Multiplication

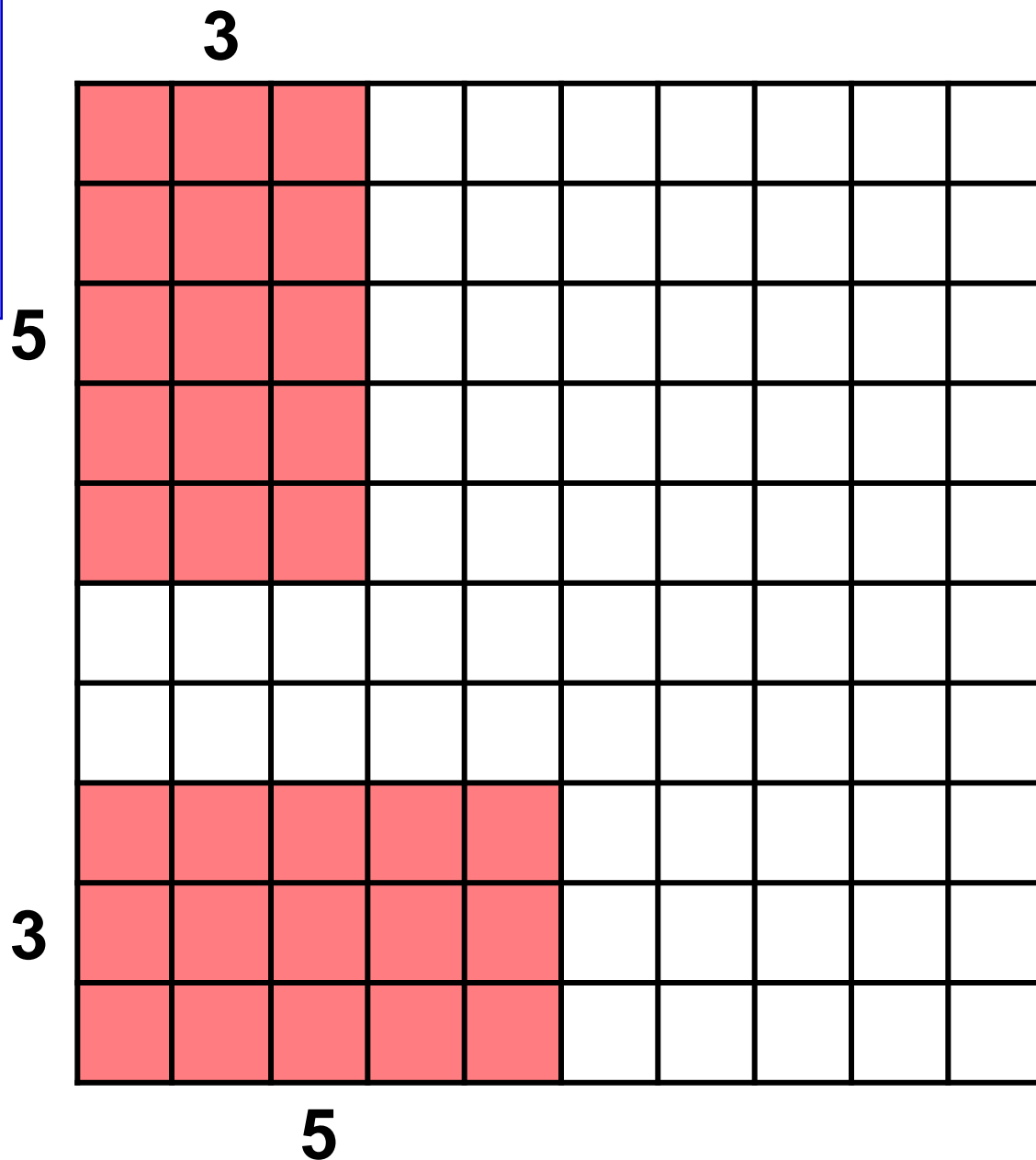


**Linking
Multiplication
Facts and
Geometry**

$$5 \times 3$$

equals

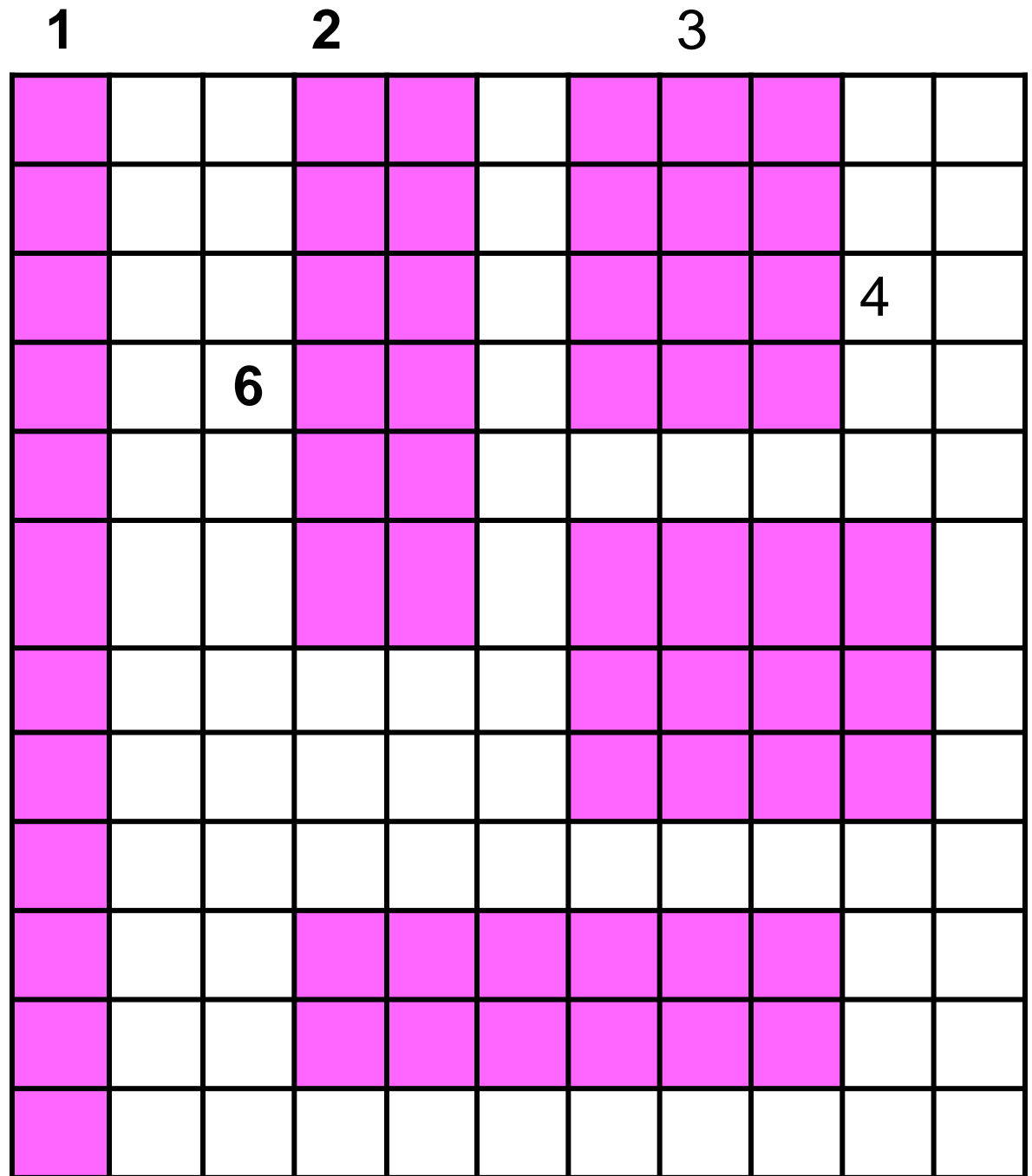
$$3 \times 5$$



Linking Multiplication Facts and Geometry

What are the
factors of 12?

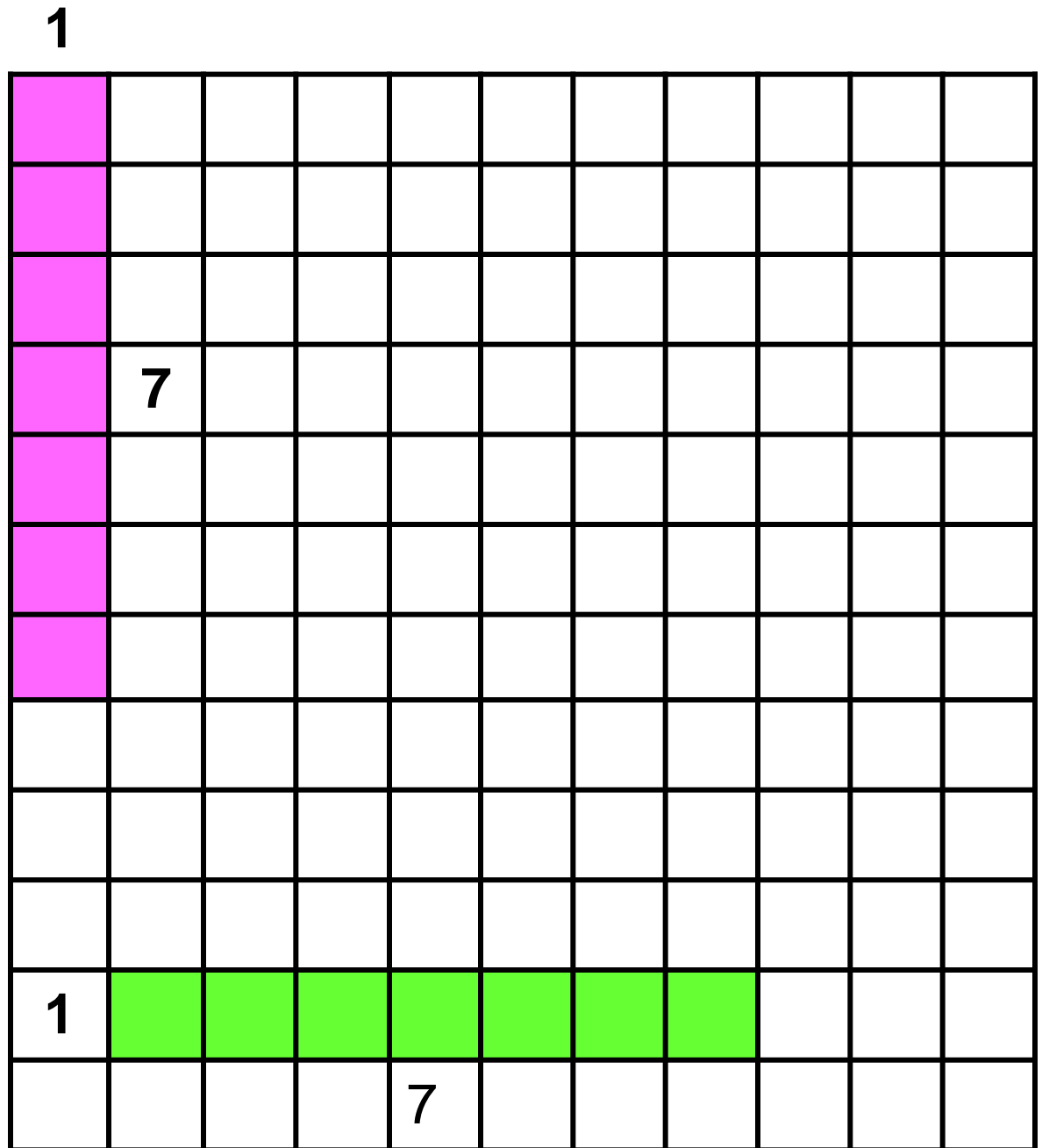
How many
different
rectangles
can be made
from 12
squares?



Linking Multiplication Facts and Geometry

What are the
factors of 7?

How many
different
rectangles
can be made
from 7 unit
squares?

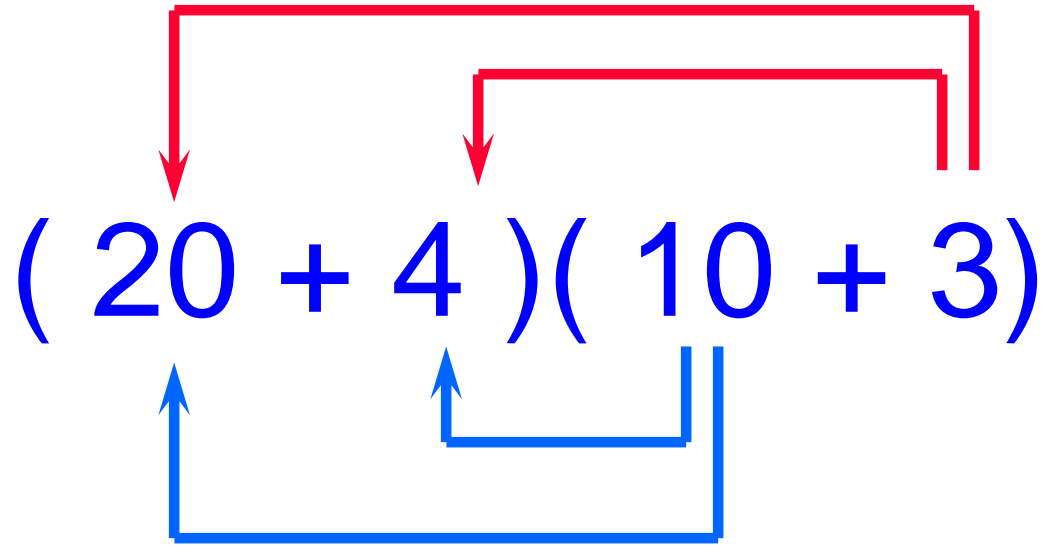


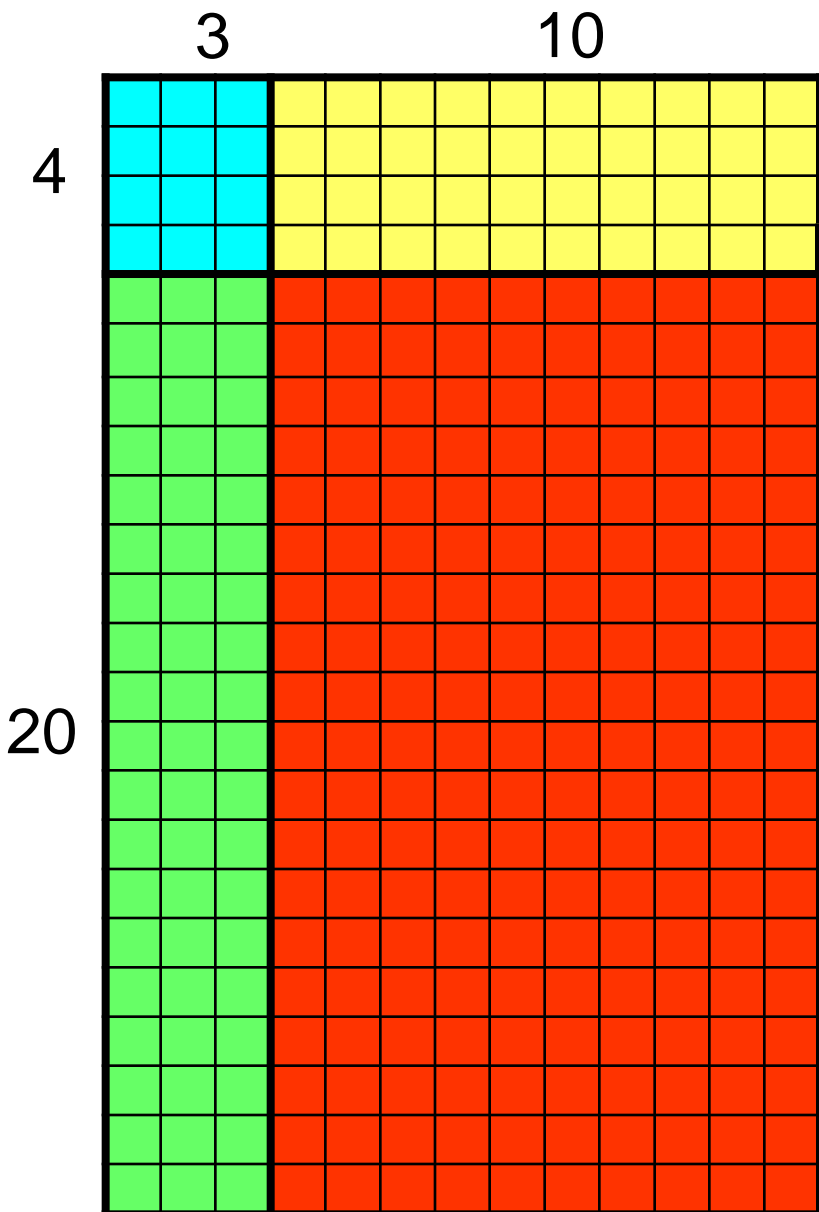
Is the “Standard” Algorithm Just a Set of Rote Procedures?

$$\begin{array}{r} \overset{1}{24} \\ \times 13 \\ \hline 72 \\ 24 \\ \hline 312 \end{array}$$

An Alternate Algorithm?

$$\begin{array}{r} 20 + 4 \\ \times \underline{10 + 3} \\ 12 \\ 60 \\ 40 \\ + \underline{200} \\ 312 \end{array}$$

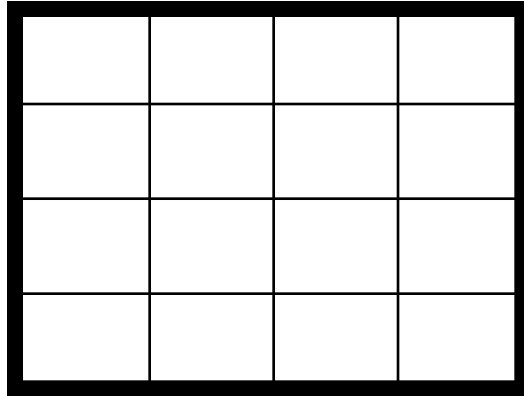




The Link to Area

$$\begin{array}{r}
 24 \\
 \times 13 \\
 \hline
 12 \\
 60 \\
 40 \\
 + 200 \\
 \hline
 312
 \end{array}$$

The Mirror Problem



Parts

Corner



Edge

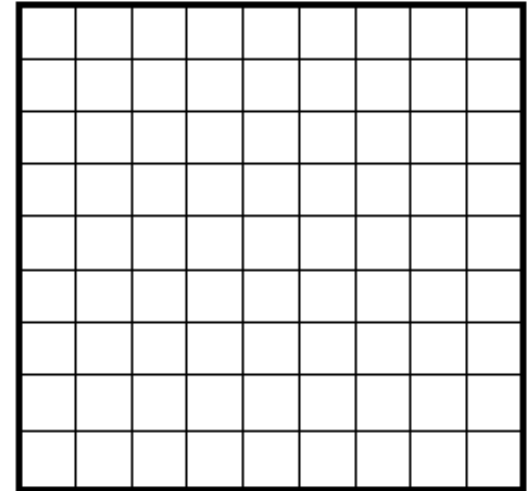
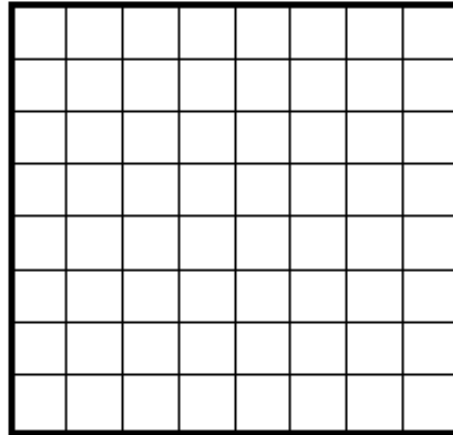
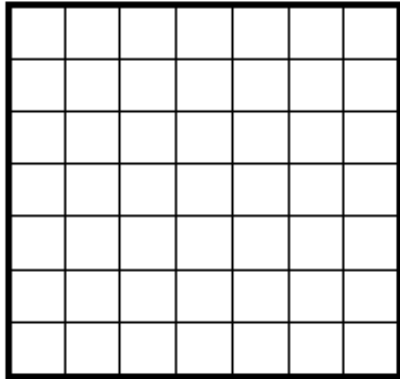
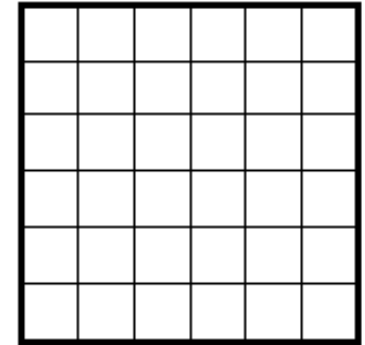
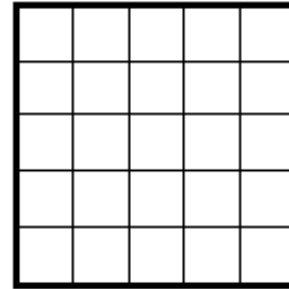
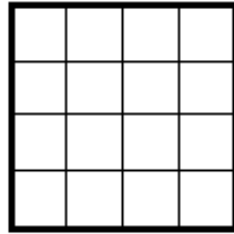
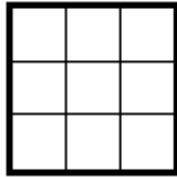
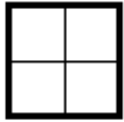


Center

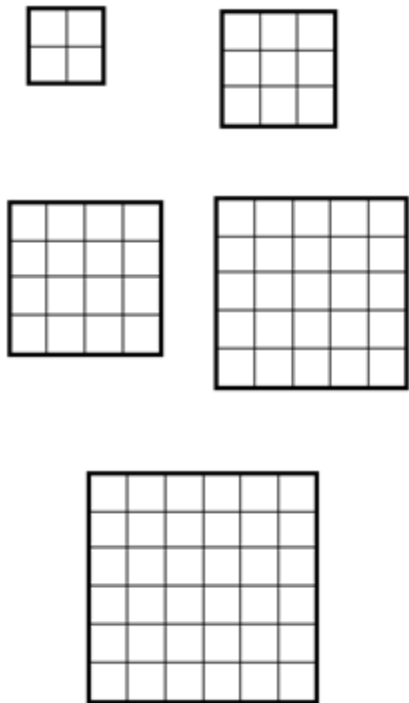





A company makes “bordered” square or rectangular mirrors. Each mirror is constructed of 1 foot by 1 foot square mirror tiles. The mirror is constructed from the “stock” parts. How many Tiles of each of the following stock tiles are needed to construct a “bordered” mirror of the given dimensions?

The Mirror Problem



The Mirror Problem



Mirror Size	Number of 2 borders tiles 	Number of 1 border tiles 	Number of No border tiles 
2 ft x 2 ft	4	0	0
3 ft x 3 ft	4	4	1
4 ft x 4 ft	4	8	4
5 ft x 5 ft	4	12	9
6 ft x 6 ft	4	16	16
7 ft by 7 ft	4	20	25
8 ft by 8 ft	4	24	36
9 ft by 9 ft	4	28	49
10 ft x 10 ft	4	32	64

The Mirror Problem

Mirror Size	Number of "Tiles" (2 borders)	Number of "Tiles" (1 border)	Number of "Tiles" (No borders)	Total Number of "Tiles"
2 ft x 2 ft	4	0	0	4
3 ft x 3 ft	4	4	1	9
4 ft x 4 ft	4	8	4	16
5 ft x 5 ft	4	12	9	25
6 ft x 6 ft	4	16	16	36
7 ft by 7 ft	4	20	25	49
8 ft by 8 ft	4	24	36	64
9 ft by 9 ft	4	28	49	81
10 ft by 10 ft	4	32	64	100
:	:	:	:	:
n ft by n ft	4	$4(n - 2)$	$(n - 2)^2$	n^2

What Is My Rule?

$$6 @ 4 = 210$$

$$9 @ 2 = 711$$

$$8 @ 3 = 511$$

$$5 @ 2 = 37$$

$$7 @ 6 = 113$$

$$9 @ 8 = 117$$

$$10 @ 6 = 416$$

$$15 @ 3 = 1218$$

$$22 @ 14 = 836$$

$$35 @ 25 = 1060$$

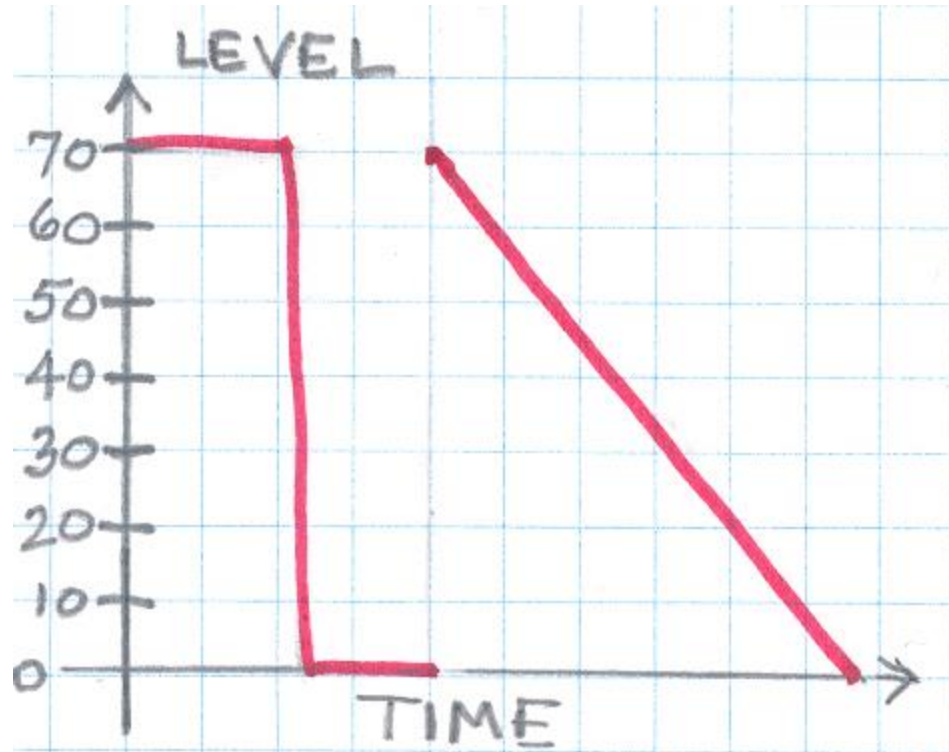
The Graph Tells a Story!

You are at the movie and you buy a cup of popcorn. The graph shows the level of the popcorn in your cup. What happened?



The Graph Tells a Story!

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Key Questions for Teachers, Schools and Districts

You Should Ask:

- Is our algebra curriculum a coherent K-12 strand?
- Is it well-articulated across grades K-12? (Do the teachers talk to each other?)
- Are students held accountable for topics studied in earlier grades? (Do teachers assess prior mastery or do they simply review or re-teach?)
- Are you talking to each other? Are you working as a team?

**“Good teaching
is one-fourth
preparation and
three-fourths theater”**

Gail Godwin, *The Old Woman*

Theater

Five essentials of theater are:

- the actors (*students and teachers*),
- the vehicle (*content, curricula, materials*)
- the audience (*parents, community, workplace*), and
- the setting (*classroom and the world around us*).
- The final essential is the performance (*achievement*) and the actors' creative work (*learning and growth*).

Invite, Not Motivate

**“A great teacher never strives
to explain his vision.
He simply invites you to stand
beside him and see for
yourself.”**

Rev. R. Inman

$$8Q + 2Q = ?$$

email: JRubillo@verizon.net