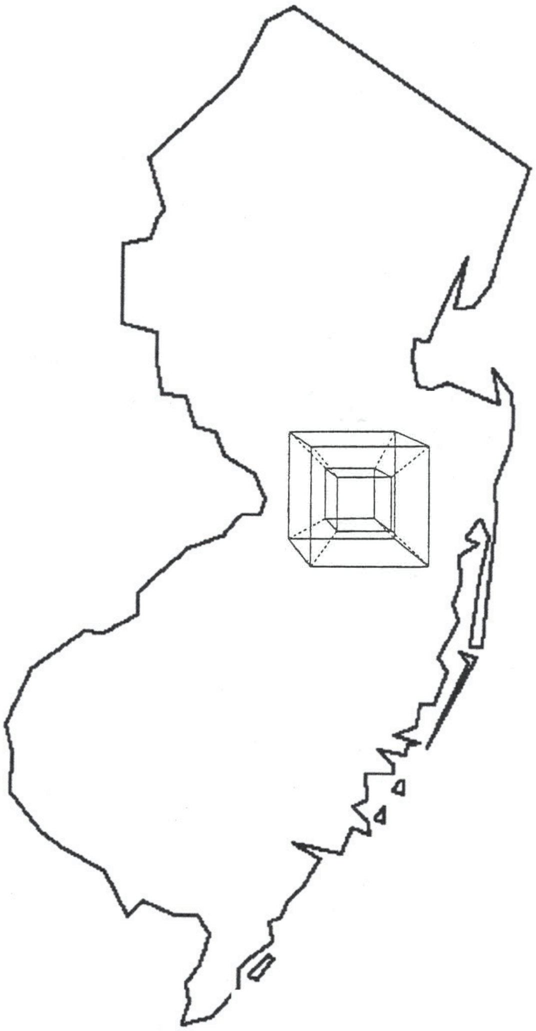

The New Jersey Mathematics Teacher

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Association of Mathematics Teachers of New Jersey

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PAGE ARTICLE

EDITORIAL

A Presidential Issue

Dr. Thomas P. Walsh, Editor-in-Chief

We are particularly proud to offer this issue of The New Jersey Mathematics Teacher. In this issue are articles by four distinguished past presidents.

Our first presidential article was written by Dr. Janice-Lynn Schuhan and Dr. Max Sobel. Dr. Schuhan is a mathematics teacher in Belleville High School. At the same time she did her doctoral work under Dr. Sobel, and she mentored a student teacher I supervised in the spring of 2010. At the same time she was mentoring him, she coaxed Dr. Max Sobel (the mathematics educator we have named our highest honor after) into writing, and collaborating with her in an article about fostering problem-solving during the summer months. Don't wait for the summer to implement the ideas in this article, however. It can (and should) be used by students during any vacation, or during their lunch breaks, or any spare time they have. As Howard Gardner suggests, it offers wonderful strategies for fostering problem-solving, and for entering a problem through multiple entry points.

Our second presidential article explains mathematical poetry by explaining what it is *not.* It gives many examples of the kinds of mathematical poetry that exist, and discusses the mathematical content and wordplay mathematical poetry can illustrate. It encourages readers to write their own poetry. It is erudite: we expect that of Dr. John Hammett of St. Peter's College.

Our third presidential article is one by our NJ math couple: Dave and Joyce Glatzer. They have offered a piece that is a wonderful twist on traditional lessons (not that any of you are traditional). They give suggestions on how to turn traditional exercises into inquiry-based exercises, by asking non-traditional questions about the exercise.

We are also pleased to present Dr. Welder, who explores the role of personality in performance in a mathematics course, and Dr. Clayton, who discusses the importance of questioning techniques in a mathematics class and the importance of putting those questions in one's lesson plan for the day.

Enjoy this issue, and as always, please be welcome to write to me if you have something to say. I can be reached at tpwalsh@kean.edu

All in the Family!

David J. Glatzer Mathematics Consultant and

Joyce Glatzer Mathematics Supervisor, K-6, West New York, NJ School System

Past Presidents Dave and Joyce Glatzer offer a classroom idea that is very easy to implement in many different contexts. This idea allows a teacher to transform many different textbook exercises into inquiry-based questions; problem solving opportunities for students. These ideas address both the NCTM standards of Number and Operations, Problem Solving, Representation, and Communication.

Tom Walsh. Editor – NJ Math Teacher

Textbook and/or teacher directed mathematics lessons include exercises related to the lesson objectives. These exercises are typically used for guided practice and for independent practice (homework). In most cases, the exercises are not related to one another. As a result, students generally think of them as isolated practice with little connection.

This article offers an alternative to exploit the relationship among the exercises within a lesson. It is expected that these relationships will help solidify the learning. Moreover, students may benefit from opportunities to use critical thinking, estimation, and number sense.

The sample exercises below are all based on consideration of one specific multiplication problem:

54 x 22 =

1. Without doing the multiplication, how would you know that the product is over 1000?

2. Make up two division problems that result from the above multiplication.

3. Without doing any computation, answer each of the following by using the above multiplication example.

5.4	54	5.4	540	540	220
x 2 2	x 2.2	x 2.2	x 22	x 220	x 54

4. What happens to the product in the original multiplication example, if one factor is decreased by 50% (or one factor was halved) and the other factor remains the same?

5. What happens to the product in the original multiplication example, if each factor is decreased by 50%?

6. Why does 27 x 44 give the same product as the original example?

7. An auditorium has 22 rows of seating. Each row has 54 seats. How many total seats are in the auditorium?

8. If the area of a rectangle is 1188 square inches and the length is 54 inches, how long is the width?

9. A t-shirt sells for $ 5.40. How much will it cost to buy 22 t-shirts?

It is important for students to see how these exercises are related and that they are not simply isolated or random questions. In most of the exercises, the "answer" is right there for the taking. As teachers structure lessons and assignments, it would be beneficial for the "All In The Family" concept to be featured.

An Appreciation of Mathematical Poetry Begins with What It Isn't

John E. Hammett III, Ed.D, Saint Peter's College

ABSTRACT

A first step towards appreciating mathematical poetry is recognizing what it really is not. Mathematical poetry is neither simply the elegance of the discipline nor just the mechanics of verse; it is more than an engaging recreational diversion. Though not a well defined or cohesive body of literature, mathematical poetry is nonetheless readily recognizable in its myriad forms, and readers can find meaning beyond educational and entertainment values within math poems.

What is mathematical poetry? Although a common sense reply might be to say that it is poetry involving mathematics in some manner, it is much more. Unfortunately, the discussion rapidly becomes diffuse and disorganized, because this seemingly straightforward question apparently does not have a simple answer. An exploration of the literature on math poems apparently fails to yield definitive results. Indeed, the findings at times tend to converge and at other times diverge, depending upon the source being read.

For example, consider how math poems are presented on the page, or perhaps more aptly put, represented. Some poets and other scholars interpret mathematical poetry exclusively as that which is written in a mathematical manner or with math symbols (e.g., Maslanka, 2006; Grumman, 2001). Other formats would likely not be viewed as mathematical poetry from their perspective. Meanwhile, other scholars and poets would seem to disagree, opting not to utilize expressions that are primarily symbolic but choosing instead to employ presentation styles that are primarily verbal and in a more traditional poetic format. Ganz (2010) catalogues a number of such verbally rich math poems, some of which are indeed best suited for readers who have pursued advanced study in the field of mathematics. In particular, the poems that she cites concerning contemporary mathematics, such as those addressing Fermat's Last Theorem and the Riemann Conjecture, are eloquent and conceptually sophisticated without dwelling on symbols.

Still other math poems blend both forms of representation, symbolic and verbal. The mathematician Charles Dodgson, better known by his pen name, Lewis Carroll, wrote this poem, as reprinted by Eves (1971), and expresses itself in part with some very special symbols: a quadratic equation.

> Yet what are all such gaities to me
>
> Whose thoughts are full of indices and surds?
>
> $x^2 + 7x + 53 = 11/3$

Next, consider the role that math poems seem to play, and the apparent functions they tend to serve in their very existence. A survey of the literature and pertinent collections of such poetry has revealed the following abridged list of classifications,

- Symbolic expressions;
- Literary artworks;
- Adaptations of or farcical imitations of existing poems;
- Illustrations of mathematical concepts;
- Accolades honoring mathematicians;
- Puzzles and other rhyming riddles (*e.g.,* classic recreational mathematics);
- Mnemonic expression, rhyming rules and reinforcements;
- Story problems set to verse (*i.e.,* poem problems);
- Problem solutions written in verse answering questions posed in prose; and
- Song lyrics.

Some poems fit compactly into one category while others overflow into two or more, and some of the categories have sub-categories. [Selected details will follow subsequently.] Suffice to say, other differences (e.g., mechanics, format and style) exist as well. All these deviations and more among mathematical poetry tend to leave the literary art form that might seem superficially easy to spot on the printed page but in actuality is destined to defy all reasonable attempts at definition or characterization.

However, despite this apparent lack of a unifying description, several themes do emerge and can provide some illumination while still not specifically addressing the initial query (i.e., what is mathematical poetry?) in its entirety. These themes can be synthesized into several constraining ideas that enable readers to develop an appropriate conceptualization of mathematical poetry without actually defining it. The claim, therefore, is that understanding what a math poem is first begins by knowing what it is not, or what it doesn't necessarily need to be. In other words, mathematical poetry is perhaps more readily defined by what it is not rather than what it is. In keeping with the style and spirit of math poems, the statements are framed mathematically.

Statements #1:

Mathematical Poetry ≠ the Poetry of Mathematics

Mathematical Poetry ≠ the Mathematics of Poetry

Statement #2:

$\{x|x$ is a Math Poem$\}$ is not a well-defined set.

Statement #3:

{Mathematical Poetry} ≠ {Recreational Mathematics}

Thus, these three summary statements place in perspective that which mathematical poetry is *not*, hence narrowing the remaining solution region or feasible set of that which is math poetry, to use an appropriate mathematical metaphor here.

Statements #1:

Mathematical Poetry ≠ the Poetry of Mathematics

Mathematical Poetry ≠ the Mathematics of Poetry

Again, the argument here is that, in order to illuminate what mathematical poetry *is*, that which it *is not* must be considered. Therefore, by way of the following double negative definition: *math poetry* is neither, or at least not necessarily anyway, *the mathematics of poetry* (*e.g.,* rhyme scheme, meter) nor *the poetry of mathematics* (*i.e.,* the elegance or beauty of the discipline). Although mathematics itself can be viewed as poetic in nature (*e.g.,* Guillen, 1995; Osserman, 1995), that construct alone does not seem sufficient to label mathematical ideas as poems. Similarly, mathematical poetry is more than the mechanics and technical components of the verses and rhymes of poetic writing.

The right-hand-side of the first inequality in the verbal system of Statement #1 (*i.e.,* mathematical poetry is not just the poetry of mathematics) refers figuratively to the apparent intrinsic elegance found in a proof or other refined, distilled solution to some mathematical problem or question. Consider the notion that mathematics itself is poetic in how it occurs naturally (*i.e.,* in nature). Equations could be viewed as artistic expression with a special language, the language of mathematics. Guillen (1995) said that, "*mathematics* is arguably the most successful global language ever spoken" (p. 2). He continued,

> In the language of mathematics, equations are like poetry: They state truths with a unique precision, convey volumes of information in rather brief terms, and often are difficult for the uninitiated to comprehend. And just as conventional poetry helps us to see deep *within* ourselves, mathematical poetry helps us to see *beyond* ourselves – if not all the way up to heaven, then at least out to the brink of the visible universe. (p. 2)

In particular, he is referencing landmark equations[1].

Guillen continues,

> While the [five] equations [that changed the world] represent the discernment of eternal and universal truths, however, the manner in which they are written is strictly, provincially human. That is what makes them so much like poems, wonderfully artful attempts to make infinite realities comprehensible to finite beings. (p. 6)

He labeled these equations as "five of the greatest poems ever inspired by the human imagination" (p. 7). However, Guillen seems to have taken, although perhaps justifiably so, some poetic license, pardon the pun, in deeming these equations as math poems. Once again, while these equations are most certainly organically beautiful, poetic and mathematical, they <u>are not really</u> consistent with more broadly based

1 Isaac Newton's Universal Law of Gravity, Daniel Bernoulli's Law of Hydrodynamic Pressure, Michael Faraday's Law of Electromagnetic Induction, Rudolf Clausius's Second Law of Thermodynamics, and Albert Einstein's Theory of Special Relativity

interpretations of math poems. [In the spirit of fairness and candor, a minority perspective must be shared: Maslanka (2010) disagrees, referring to number poems, magic squares, and that which Guillen describes as Pure Math Poems.]

The right-hand-side of the second inequality in the verbal system of Statement #1 (*i.e.* mathematical poetry does not necessarily equate to the mathematics of poetry) is the premise that poems abound with many components that are mathematical, arithmetical or numerical in particular. For instance, rhyming poems contain what might be described as alphanumeric patterns. The rhyme scheme of the humorous, sometimes nonsensical, five-line limerick is a, a, b, b, and a; a conventional metric pattern to the syllabic feet contained within a limerick might be 16, 16, 11, 11, and 16 (*e.g.,* Deutsch, 1982), though certainly variations exist. The ubiquitous spiritually-themed poem of Japanese origin, the three line haiku, follows a 5, 7, 5 metric; its obscure slightly longer sister verse, the five line tanka, follows a 5, 7, 5, 7, 7 syllabic pattern. Petrarchan and Shakespearean sonnets, each fourteen lines long, have different rhyme schemes for their stanzas: the Petrarchan scheme is a, b, b, a, a, b, b, a, c, d, e, c, d, and e, whereas the Shakespearean pattern is a, b, a, b, c, d, c, d, e, f, e, f, g, and g. Admittedly, not all poems have such quantitative components; for example, free verse poetry is irregular by design, and does not adhere to conventional structures of meter, length or rhyme scheme. For those poems that do possess such pattern and other numerically oriented attributes, the mere presence of said characteristics does not make poems with measurable rhythms and meters math poems.

Statement #2: The collection $\{x|x \text{ is a Math Poem}\}$ is not a well-defined set.

Realizing in response to Statement #1 that a math poem is neither the poetic elegance of math concepts nor the quantitative mechanics of poetic forms, then what else is mathematical poetry or again, what is it not? Though readily recognizable to some readers, math poems are not really a cohesive whole. Regrettably, math poets and other scholars do not entirely agree about what constitutes a math poem.

Some scholars and poets seem to limit math poetry only to that which is written symbolically, in what might be described as a mathematical manner. In other words, only poems which employ mathematical representations and operations are recognized as math poems by those that adhere to this perspective. Poems written or read under this strict interpretation of mathematical poetry would likely contain what Munroe (1963) refers to "the language of mathematics … [or] Mathese" (p. 7). According to some information that poet and former math professor JoAnne Growney (2010) posted on her math poetry blog, "mathematical language can heighten the imagery of a poem; mathematical structure can deepen its effect." She also notes that such "poems [can be] made rich by mathematical ingredients."

American Poetry, or the *Poetry of the United States*, has been classified by the Academy of American Poets (2010) into 31 Schools and Movements, spanning in reverse alphabetical order from Symbolism to Acmeism, noting that the symbolist poetry in this context refers to those verses that evoke images that substitute for more mundane constructs and not to the employment of math symbols in the poetry. Alternatively, Holman (2001) divides *American Poetry* into eight Schools[2], one of which is Pluraesthetic, otherwise known as Polyaesthetic. Holman further dissects Pluraesthetic Poetry into seven Sub-Schools[3], one of which is Mathematical Poetry.

Math poems can be written in symbolic form with at least two different approaches: these compositions that contain overt mathematical characters can be *either* numerical expressions that employ math symbols in some poetic pattern *or* minimalistic constructions, abbreviated in appearance and configuration while conveying some non-quantitative essence.

An example of a numerically expressed piece is *A Math Poem*, by John Saxton, source unknown:

This arithmetic statement adheres to the general

2 The other seven Schools of American Poetry, according to Holman (2001) are Mainstream Poetry, Easy-Stream Poetry, Language or Acadominant Poetry, Contra-Genteel Poetry, Neoformalist Poetry, Infra-Verbal Poetry, and Hypertextual Poetry

3 The other Sub-Schools are Visual Poetry, Sound Poetry, Performance Poetry, Flow-Chart Poetry, Compucentric Poetry, and Polylingual Poetry

poem pattern known as a limerick, and it can be read as follows:

A Dozen, a Gross, and a Score
Plus three times the square root of four,
    Divided by seven,
    Plus five times eleven,
Equals nine squared and not a bit more.

Another symbolic math poem, concocted by Betsy Devine and Joel E. Cohen (1992), can be expressed as follows, and also reads like a limerick:

$$\int_{1}^{3\sqrt{3}} (z^2\, dz)\ \ (cos(3\pi\ /\ 9)) = ln\ \sqrt[3]{e}$$

This poem can be read as follows:

Integral z squared dz
From one to the cube root of three
    Times the cosine
    Of three pi over nine
Equals log of the cube root of *e*.

Both of these symbolically expressed limericks are mathematical equations that can be verified as accurate. These types of poems include apparently equivalent expressions that can be confirmed (e.g., the Saxton poem), operations that can be performed (e.g., the Devine and Cohen poem), or equations that can be solved algebraically (e.g., the previously-listed Dodgson poem). They engage the reader in computation and calculation.

Pluraesthetic Poetry can also be considered Minimalistic or MNMLST Poetry (Grumman, 2001). For example, Grumman cites the following work by LeRoy Gorman.

the birth of tragedy

$(! + ?)^2$

Grumman interprets this minimalist poem as follows" tragedy equals a blur of exclamations and questions multiplying against each other." (p. 10) Grumman also cited his own work, a variant on the traditional haiku, at least in appearance.

Mathmaku #2

March=
(meadows.)(:/.),
slowly

His interpretation of his own minimalist poem is as follows: "… the effect of winter on the natural world, as represented by meadows, is similar to the effect of a period on a sentence: it stops it. … [T]he change of winter to spring … has the inevitability and cleanness of mathematics (in slow motion)." (p.11) This second minimalist poem is also an example of equational poetry, which is defined by Maslanka (2010) as "an artistic expression created by performing mathematical operations on words or images."

These forms of math poems blend the symbolic and the artistic into one form. Although math poets that generally adhere to what might be labeled as minimalist math poetry would likely fail to acknowledge or recognize, or at best rapidly dismiss, other forms as math poems (*e.g.,* Maslanka (2010) succinctly acknowledges lexical forms of math poems but quickly shuttles interested readers elsewhere before returning to his preferred formatting), they themselves do not always agree. Heated exchanges can occur among these poets over, for example, the role of the equal sign in math poetry (Maslanka, 2010).

Symbolically expressed math poetry seems especially prone to continued evolution. A new subfield of mathematical poetry is called oulipian[4] which "produced, among other works, Queneau's outlandish book: Cent Mille Milliard de Poèmes, which indeed offers the reader one hundred trillion (10^{14}) poems." (http://www.nous.org.uk/oulipo.html, June 24, 2010).

Another recent variant is the **S+7** Method (*e.g.,* Growney, 2006; Marie & Reggiani, 2007) which supplants every substantive word in a piece of literature with the seventh subsequent word from a chosen dictionary.

What about other formats and types of math poetry? A vast array exists within the traditional verbal or lexical style of presentation, as Maslanka (2010) might describe it. Perhaps the most famous and well-known

4 The name of this contemporary movement, comprised mainly of French speaking poets, is loosely derived from a French language acronym for the Workshop of Potential Literature, or OULIPO.

piece of mathematical poetry is the nursery rhyme entitled, "As I Was Going to St. Ives." Admittedly, the math involved in this poetic problem is suspect, since the common answer hinges upon a technicality, namely that only one person, the narrator, heads toward St. Ives while the rest of the people and animals encountered are presumably heading away from it.

Consider another nursery rhyme entitled, "How Many Miles to Babylon?"

How many miles to Babylon?
Three score and ten.
Can I get there by candlelight?
Aye, and back again.
If your feet are nimble and light,
You'll get there by candlelight.

In a section of text intriguingly labeled, "Tilts and Tourneys," Albert Beiler (1966, p. 298) provided his own version of a familiar nursery rhyme where the hot cross-bun man cried:

Hot cross-buns, hot cross-buns,
  One a penny, two a penny, hot cross-buns.
If your daughters don't like them
  Give them to your sons!
Two a penny, three a penny, hot cross-buns,
  I had as many daughters as I had sons,
So I gave them seven pennies
  To buy their hot cross-buns."

How many children were there if they were all treated alike and if there was only one way in which to purchase the buns?

Henry Dudeney (1967, p. 214) included the following alleged tombstone inscription in his collection of mathematical recreation, a field of mathematics that will subsequently be discussed in this article.

An Epitaph (A.D. 1538)

Two grandmothers, with their two granddaughters;
Two husbands, with their two wives;
Two fathers, with their two daughters;
Two mothers, with their two sons;
Two maidens, with their two mothers;
Two sisters, with their two brothers;
  Yet only six in all lie buried here;
  All born legitimate, from incest clear.
  How might this happen?

Of the 536 problems in this book, this was the only one formatted as a poem.

Consider the following examples of math poems that stand as pieces of literary artwork separate from their import within mathematics education. For example the following poem by A.C. Orr (Eves, 1971, p. 28) sings accolades to an "immortal" mathematician.

Now I, even I, would celebrate
In rhymes inapt, the great
Immortal Syracusan, rivaled nevermore,
Who in his wondrous lore,
Passed on before,
Left men his guidance how to circles mensurate.

This poem, besides being a masterful piece of literary writing, is also a mnemonic for the first 31 digits of the irrational number, π: 3.1415926535897 93238462643383279 .... It is also a lyrical tribute to the Great Immortal Syracusan, Archimedes, using the characterization made popular by Rollin (1839) in his vast tome on ancient history. Similarly, Lewis (1983) cites a prize winning mnemonic that also suggests the same digits of that transcendental number by way of a poem with rhyming couplets:

Now I will a rhyme construct,
By chosen words the young instruct,
Cunningly devised endeavor,
Con it and remember ever,
Widths in circle here you see,
Sketched out in strange obscurity.

Keith (2001, personal communication) describes the piku, a variation on the traditional haiku, as another form of mnemonic math poetry that accomplishes a portion of the same task as the previous two poems: each word sequentially signifies a corresponding digit in π. The piku otherwise follows the line and syllable requirements of the haiku. The following is an example written by Keith (2001):

It's a bird, a plane
appearing in cloudy skies --
It's brave superman.

Returning to the tribute format, consider this piece about the influential Ancient Greek mathematician entitled, “Euclid,” by the poet Vachel Lindsey (http://www.katherinestange.com/mathweb/index.html).

Old Euclid drew a circle
On a sand-beach long ago.
He bounded and enclosed it
With angles thus and so.
His set of solemn graybeards
Nodded and argued much
Of arc and of circumference,
Diameter and such.
A silent child stood by them
From morning until noon
Because they drew such charming
Round pictures of the moon.

JoAnne Growney wrote a 63-line poem as a tribute to Emmy Noether entitled, “My Dance is Mathematics.” (http://www.katherinestange.com/mathweb/index.html) She refers to the distinguished mathematician as “a poet of logical ideas,” and writes,

Emmy Noether’s abstract axiomatic view
Altered the face of algebra.
She helped us think in simple terms
That flowered in their generality.

A repeated line echoes the title,

If a woman’s dance is mathematics,
Must she dance alone?

This poem, “Ode to the Golden Rectangle,” was written by Keith Ferland (1991, p. 118), paying tribute not to a person but to an elegant mathematical construct.

Oh polygon, thy shape divine.
Pure in beauty, a simple line,
da Vinci, Dürer fell under your spell,
Ancient Greeks to modern man as well.

A proportion truly natural,
A length that is constructible.
And frisky rabbits go on and on
Unaware of the regular pentagon.

Oh golden one, how you do dare
To humble even the perfect square.
And now you outdo the famous jocks
Since your shape *is* the Wheaties box!

The poem, “Pascal and the Parabola,” (Robson & Wimp, 1979, p. 53) is a picture poem that crafts the poem into a shape reinforcing the theme in an instructional and informative manner.

Thinking, frail reed,
of you so easily bent
and broken but not always knowing it,
I imagine how you shivered
at the thought of far-fetched
vectors, curves and conic sections
slicing planes you occupied
so queasily, caught and propped
between two chairs so as not to slip
and fall unwittingly through
yawning gaps in the parquetry,
to glide between the beams and on
through bedrock, mantle, iron core
and out again through green
antipodes, a rocketing parabola
streaking out toward yet
unsprung infinite maws.

Benjamin Banneker wrote in the late eighteenth century what he likely considered a riddle, but what perhaps more aptly fits as one of the first true story problems set to verse in human history, “The Puzzle of the Cooper and the Vintner.” (Eves, 1971, p. 140-141)

A cooper and a vintner sat down for a talk,
Both being so groggy that neither could walk;
Says cooper to vintner, “I’m the first of my trade,
There’s no kind of vessel but what I have made,
And of any shape, sir, just what you will,
And of any size, sir, from a tun to a gill.”
“Then,” says the vintner, “you’re the man for me.
Make me a vessel, if we can agree,
The top and the bottom diameter define,
To bear that proportion as fifteen to nine,
Thirty-five inches are just what I crave,
No more and no less in the depth will I have;
Just thirty-nine gallons this vessel must hold,
Then I will reward you with silver or gold --
Give me your promise, my honest old friend.”

"I'll make it tomorrow, that you may depend!"
So, the next day, the cooper, his work to discharge,
Soon made the new vessel, but made it too large;
He took out some staves, which made it too small,
And then cursed the vessel, the vintner, and all.
He beat on his breast, "By the powers" he swore
He never would work at his trade any more.
Now, my worthy friend, find out if you can,
The vessel's dimensions, and comfort the man!

As yet another example of how math poems might exist, the solution to a story problem, known as Knot I, was credited by Carroll (1958, p. 83) to two individual, as acknowledged by the pseudonyms, Simple Susan and Money Spinner. The original expression of the problem was in prose; this solution was a poem.

Given all the different nuances to mathematical poetry, is any consensus possible? Some have tried. Maslanka (2010) codifies mathematical poetry into four forms: (1) Mathematics Poems, which can be traditional verbal or lexical poetic expressions that deal with or are influenced by mathematical ideas; (2) Mathematical Visual Poetry, which can be poems that take form geometrically (e.g., verbogeometric); (3) Equational Poetry, which can use mathematical operations and symbols for expressive purposes; and (4) Pure Maths Poetry, which can be the previously discussed and modestly controversial view that mathematics itself can be viewed as expressions of poetry.

Growney (2006) offers perhaps the most succinct, arguably more mainstream, distillation, identifying two types of math poems: those with mathematical imagery and those with mathematical structure. However, she astutely does not claim that these are the only forms that exist. She concludes her synthesis of math poetry with this observation, "There is no way to end a consideration of the links between mathematics and poetry. They go on and on."

While mathematical poetry is not necessarily well-defined, it is certainly sufficiently comprehensible that some noteworthy poets have penned math poems. As previously mentioned, Lewis Carroll, also known as Charles Dodgson, was one. In addition, the following stanza is attributed, source unknown, to the famous English author, Chaucer, circa 1390.

It was four o'clock according to my guess.
Since eleven feet, a little more or less,
My shadow at the time did tell,
Considering that I myself am six feet tall.

More contemporarily, Wallace Stevens (1916) incorporated references to geometric figures such as ellipses, rhomboids, and right-angled triangles in "Landscape VI from 'Six Significant Landscapes'." In "Euclid alone has looked on Beauty bare," Edna St. Vincent Millay apparently idealizes the ancient Greek mathematician's perspective on the world. (http://www.katherinestange.com/mathweb/index.html) Carl Sandburg (1993) wrote a poem simply titled, "Arithmetic." It ends as follows:

> If you ask your mother for one fried egg for
> breakfast and she gives you two fried
> eggs and you eat both of them, who
> is better in arithmetic, you or your
> mother?

Wilkins (undated) aptly reflected that, "Poetry is NOT meter. ... Poetry is NOT alliteration [which is a pattern, again mathematical]. Poetry is NOT rhyme. Poetry is NOT any one convention." This perspective absolutely applies to mathematical poetry; therefore, math poems will likely remain not well-defined but rather they will remain engaging, provocative, and evolutionary. Therefore, despite the lack of definition surrounding the admittedly amorphous genre and its various collections of mathematical poetry, the premise of the math poem is sound.

Statement #3:

$$\{\text{Mathematical Poetry}\} \neq \{\text{Recreational Mathematics}\}$$

Before commencing this portion of the discourse, elaborating upon the field of mathematical recreations seems reasonable. Jones (1932) considered mathematical recreations both a "form of mental diversion" and "a mental tonic for eliminating the 'sleepy brain' by sweeping away the cobwebs." (p. x) According to Gardner (1979),

> One can define 'mathematical games' or 'recreational mathematics' by saying it is any kind of mathematics with a strong play element, but this is to say little, because 'play,' 'recreation,' and 'game' are roughly synonymous. In the end one has to fall back on such dodges as defining poetry as what poets write…. Recreational math is the kind of math that recreational mathematicians enjoy.

> Although I cannot define a mathematical game any better than I can a poem, I do maintain that, whatever it is, it is the best way to capture the interest of young people in teaching mathematics. A good mathematical puzzle, paradox, or magic trick can stimulate a child's imagination much faster than a practical application (especially if the application is remote from the child's experience)… (p. xi).

Note that coincidental comparison between recreational mathematics and poetry. He continues,

> Not only children but adults can become obsessed by a puzzle that has no foreseeable practical use, and the history of mathematics is filled with examples of work on such puzzles, by professionals and amateurs alike… (p. xii).

Bakst (1954) said,

> Mathematical recreations have a special place in the mathematical literature. They also have a special place in mathematical pursuits. Generally, recreation is a diversion, or a digression from those activities which represent everyday work, whatever form this work may take. A recreational activity is supposed to be in a lighter vein. … the more or less flippant. Thus, we find relaxation in a jest or joke. Some have, however, the mistaken notion that mathematics does not lend itself to relaxation. (p. v)
> Mathematical recreation … is a mathematical approach to a situation which involves the curious, the strange, the unusual, and often the seemingly impossible. Mathematical recreation may be thought of as permitting mathematics to let down its hair. This, however, does not mean that mathematical principles are sacrificed. Such principles are meticulously preserved, although they are simplified. Thus, the bite, which frightens most, is painlessly removed. (p. v)

Gardner (1978) said, elaborating on recreational mathematics, "Word play is just like mathematics. The symbols are letters and words." (p. 140) Gardner (1978) defines a rebus is a "picture that represents words in some puzzling way" (p. 150). Here are some examples he cites:

BIS ECT

PE RI OD IC

E^{x}PONENT

Incidentally, Minimalist poets might approve of these representations as math poems. Northrop (1944) noted, "… a [mathematical] paradox is anything which offhand appears to be false, but is actually true; or which appears to be true, but is actually false; or which is simply self-contradictory" (p. 2). He cites what he describes as a "well-known riddle," (p. 16)

> Brothers and sisters have I none,
> But that man's father is my father's son

He goes on to note that "such [family relations] puzzles are not, strictly speaking, a part of mathematics …" (p. 16). However, he points out that these puzzles apparently require some thought processes often associated with mathematicians in order to solve them.

As yet another example, Gardiner (1987) described alphametics as word sums. He affirms that such problems promote pattern recognition and identification as well as an "active interpretation of a problem whose statement" (p. 5) is non-routine. He finds this useful for both children and adults. Hunter and Madachy (1963) defined an alphametic as "a mathematical problem in which the numbers are replaced by letters or words that form sensible words or phrases" (p. 243).

While there certainly is more to the field of mathematical recreation than math poems (e.g., Bolt (1992) posed 131 problems in <u>Mathematical Cavalcade</u>, but none as poems; in <u>Mathematical Quickies</u>, Trigg (1967) posed 270 problems, but none as poems), not all math poems are recreational mathematics. As indicated in Statement #3, mathematical poetry is not synonymous with the field of Recreational Mathematics because not all math poems are mental diversions or educational entertainment. Some transcend those manifestations, as engaging and stimulating as they are, and become much more.

Make no mistake: mathematical poetry has not gotten spotlight attention on any center stage in the history of mathematics. Indeed, the place of a number of math poems is relegated to the periphery, as subtly noted by the headings of chapters and text sections that contain them. Pickover (1995) reprinted song lyrics "as an example of recursion" (p. 207) under the heading, "Digressions." Beiler (1966) adapted a nursery rhyme as a math problem (p. 268) under the heading, "Tilts and Tourneys." The chapters of Jones' (1932) book on mathematical recreation extend from its colorful title: Mathematical Nuts for Lovers of Mathematics. Headings include "Nuts for Young and Old," "Nuts for the Fireside," "Nuts for the Classroom, and "Nuts for the Professor." Still, despite the unfortunate relegation of math poems to the side show, continuing the big top metaphor, the mathematical poetry of recreational mathematics is not a recent educationally entertaining novelty act. For example, as previously mentioned, some traditional nursery rhymes, which date back countless centuries, are at their heart mathematical questions crafted as logic puzzles.

Returning to Statement #3, while some math poems can indeed be characterized as diversions or entertainment, saying that all mathematical poetry belongs in this specialized field would be an injustice to those that are, for example, pure art forms or depictions of important mathematical conceptions within them.

In conclusion, given that mathematical poetry is not limited to (a) the lyrical beauty and elegance of math or the mechanics of verse, and given that it isn't especially well-defined as a genre, and given that all math poems aren't simply diversions or entertainment, even if educational, what then is the point of the math poem? Maslanka (2006) said, "Mathematics has always been used for denotation. However, … [the interpretation being advocated here] is to use math as a language for connotation." Mathematical poetry enables math to add meaning, and not just be the meaning.

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Effects of Personality on Student Performance in Collegiate Pre-Calculus

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ABSTRACT

Given the alarmingly low success rate in collegiate Pre-Calculus courses, researchers have been examining possible contributing factors. This paper seeks to fill a gap in the literature by focusing on how personality traits may influence student performance. Using the Myers-Briggs Personality Type Inventory and course exam scores, the effect of personality on collegiate Pre-Calculus performance was investigated. Groups of students (n=295) with contrasting personality traits were compared through t-tests. Students, whose characteristics were categorized as Judging (rigid, organized, opinionated), performed significantly better than those who were found to be Perceiving (curious, adaptable, open to new possibilities) (p=0.012). Differences between Judging and Perceiving traits are discussed and questions are raised regarding the potential influence of this finding on collegiate mathematics instruction.

BACKGROUND AND RESEARCH DESIGN

Almost every college or university across the United States offers a course in Pre-Calculus. Of the numerous students enrolled in Pre-Calculus classes in any given semester, some have been successful in their mathematical development and some are seeking careers in mathematics and science-related fields. Many of these students, however, have never excelled in mathematics and have no interest in pursuing a science-based career. Pre-Calculus is often taken by students merely to fulfill a general mathematics requirement for a non-mathematical degree. For many of these students, this course is the culmination of their mathematical studies and may be the last opportunity for them to be engaged in the excitement and utility of mathematical explorations. Yet, instead of encouraging further mathematical study, Pre-Calculus can be very challenging for students and even become the final barrier between a student and a college degree.

Data collected at a mid-sized land grant university in the Northwest United States, over a six-semester period, showed a Pre-Calculus success rate of only 64.67% (success defined by the university as receiving a final course grade of C- or higher). This finding highlights how important it is for college mathematics instructors to identify factors affecting student achievement in this course. Understanding what is keeping Pre-Calculus students from being successful could be the first step in finding ways to increase overall student achievement.

This study works toward that goal by investigating personality as one possible factor affecting the performance of collegiate Pre-Calculus students. Personality has been identified as having an important relationship with academic achievement in the past (Chamorro-Premuzic & Furnham, 2003; Siegel & Shaughnessy, 1992; Spray, 1995; Wagerman & Funder, 2007). However, most of the research in this area at the collegiate level has focused on mathematically advanced students (those taking Calculus I or higher). With poor success rates found in lower-level collegiate mathematics courses, the existing body of research on personality and mathematics achievement needs to be extended to investigate courses taken as part of general education curricula, like Pre-Calculus. The current study does exactly that, by testing if college students with contrasting personality traits perform differently from one another in Pre-Calculus. The research questions were designed to address Pre-Calculus performance according to each of the four scales of the Myers-Briggs Personality Type Inventory, a questionnaire based on the psychological teachings of Carl Gustav Jung (Myers, McCaulley, Quenk, & Hammer, 2003):

1. Will students classified as Introverted perform significantly better (or worse) than students classified as Extraverted?
2. Will students classified as Sensing perform significantly better (or worse) than students classified as Intuitive?
3. Will students classified as Thinking perform significantly better (or worse) than students classified as Feeling?
4. Will students classified as Judging perform significantly better (or worse) than students classified as Perceiving?

PAST RESEARCH ON PERSONALITY AND ACADEMIC ACHIEVEMENT

Most research in the field of personality and academic achievement has worked to identify connections between specific personality traits and the academic ability of students. Results from studies addressing the relationship between personality and general academic ability of high school and college students will be reviewed first. This will be followed by a short synthesis of research that specifically addresses the relationship between personality and mathematics ability, as research in this field is quite limited.

Personality and General Academic Ability

Chamorro-Premuzic and Furnham (2003) found the academic performance of British university students to be positively correlated with Extraversion (assertive and outgoing) and Conscientiousness (deliberate and careful) and negatively correlated with Neuroticism (anxious and negatively reactive to stress) and Psychoticism (not agreeable nor conscientious). This study showed that high-performing university students tended to be more extraverted, conscientious, emotionally stable, less reactive to stress, and less aggressive than the low-performing students. Similarly, Wagerman and Funder (2007) found a strong positive correlation between Conscientiousness and the academic achievement of American university students. Bratko, Chamorro-Premuzic and Saks (2006) also found Conscientiousness to be the strongest personality correlate of the overall academic achievement of high school students. However, contrary to the findings of Chamorro-Premuzic and Furham (2003), this study found high school grades to be negatively correlated with Extraversion (assertive and outgoing) and Emotional Stability (low anxiety and less reactive to stress).

Personality and Mathematics Ability

When focusing specifically on mathematics achievement, assertiveness has appeared in several research findings. Cooper, Boss, and Keith (1974) found that high school students who do significantly better in the sciences versus the humanities were assertive, reserved, and tough-minded. Odom and Shaughnessy (1989) found mathematically advanced high school students to be assertive, dominant, independent, and bold. And, Siegel and Shaughnessy (1992) found college calculus students (including those in Calculus I, II, and III) to be assertive, skeptical, and self-opinionated. Spray (1995), however, found differing results when she investigated mathematically advanced college students (with a Calculus IV-minimum background). These students were serious, cautious, socially timid, and more impersonal and distant than other college students. Contradictory to partial results of both Spray (1995) and Siegel and Shaughnessy (1992), Karnes, Chauvin, and Trant (1984) found university students' achievement on the mathematics subtest of the ACT to be significantly related to them being trusting (versus skeptical) and enthusiastic (versus serious and cautious).

METHODOLOGY

Sample

The population for this study contained all students enrolled in Pre-Calculus at a mid-sized land grant university in the Northwest United States, during one semester. The catalog description for this course reads, "Functions, graphs, and the use symbols for expressing mathematical thoughts. Polynomials, rational, exponential, logarithmic, and trigonometric functions." Students who dropped the course prior to the first exam were not considered. At the time of data collection, there were 337 students enrolled among ten sections of the course; 295 students (87.5%) participated in the study.

Measures

To determine self-assessed personality classifications, students were asked to complete the Myers-Briggs Type Inventory (MBTI) (Myers, et al., 2003). This measure was selected because it is the most widely used personality instrument in the world (Jackson, Parker, & Dipboye, 1996; Quenk, 2000), has high face validity, and is popular for use in career assessment and counseling (Pulver & Kelly, 2008).

The MBTI evaluates personality using four characteristic scales, each of which has two antipodal (contrasting) poles. The four pairs of dichotomies are as follows (Myers, et al., 2003):

- Introversion (I) – Extraversion (E): An Introverted person tends to be motivated by ideas and needs time to reflect to rebuild energy, whereas an Extraverted person draws energy from action and contact with other people

- Sensing (S) – Intuition (N): A Sensing person tends to focus on details, facts, and practical, measurable outcomes, whereas an Intuitive type is usually more interested in future possibilities
- Thinking (T) – Feeling (F): A Thinking person is more likely to base decisions upon an objective, logical analysis of facts, whereas a Feeling type is more empathetic to the situation and considers social values and relationships
- Judging (J) – Perceiving (P): A Judging type tends to focus more on planning and making immediate decisions, whereas a Perceiving person is more concerned with studying new information for more possibilities

The MBTI uses forced-choice questions to classify people based on their preferences and self-reported behaviors. The answers an individual provides are used to produce eight scores, one for each antipodal pole of the four scales. For each scale, the person is coded into the dichotomous category, and assigned its corresponding letter, based on the higher scoring pole. The interaction among these four scales creates sixteen unique MBTI personality types each designated by a different four letter combination (Myers, et al., 2003).

To evaluate student performance in the Pre-Calculus course, exam scores were collected from each of the ten class instructors. Each student had two exam scores, which were averaged to calculate an overall score for that student (based on a scale of 0-100). Although there were multiple instructors for this course, exam scores were a reliable measure of student success because content and grading variances across the multiple sections of this course were minimal. Each semester, a course supervisor designs the curriculum, schedule, and syllabus to be used in all Pre-Calculus classes at the university. Each section covers the same set of material at the same time and all students are assigned the same homework. Furthermore, all sections give identical, combined, course exams, which are written by the supervisor (with instructor input); and, a detailed grading system is employed so that all exams are graded consistently across the multiple sections.

Statistical Analysis

For each of the four personality scales, the entire sample ($n = 295$) was divided into two groups, one for each of the antipodal poles of that scale. For example, to address the first research question, the sample was separated into two groups; those who were categorized into one of the eight personality types that included an I (Introversion) were separated from those who were categorized into one of the remaining eight personality types that included an E (Extraversion). A t-test (t) was used to compare the mean Pre-Calculus scores achieved by these two groups. The hypothesis claiming that the true mean difference between the two groups' scores is equal to zero was tested against a two-sided hypothesis for a non-zero population mean difference. This process was then repeated three more times by dividing the entire sample according to the remaining three scales (Sensing-Intuition, Thinking-Feeling, and Judging-Perceiving).

Dividing the sample in four different ways resulted in unequal sample sizes (see Tables 1-4); however, the statistical process outlined above was deemed appropriate due to its robustness against unequal sample sizes. When the sample was divided using the Introversion-Extraversion, Sensing-Intuition, or Thinking-Feeling scales, homogeneity of variance was verified ($p = 0.599$, $p = 0.488$, $p = 0.682$ respectively) using Levene's Test for Equality of Variance. Therefore, the t-tests performed for research questions #1-3 were calculated under the assumption of equal variances. However, when the sample was divided using the Judging-Perceiving scale, homogeneity of variance was not confirmed ($p = 0.011 < \alpha = 0.05$). Therefore, a t-test for unequal variances was performed to answer research question #4, comparing mean scores for students categorized as Judging versus Perceiving ($F = 6.580$, $p = 0.011$, $\alpha = 0.05$).

Tables 1-4 on the following page show means and standard deviations of Pre-Calculus scores and results of the *t*-tests calculated for equality of mean scores ($\alpha = 0.05$), organized by each of the four personality scales.

Table 1

T-test for Equality of Mean Pre-Calculus Scores (α = 0.05) by Introversion (I) – Extraversion (E) Personality Scale*

Personality	n	Mean	SD	t	df	Sig. (two-tailed)	Mean Difference
Introverted Extraverted	136 159	67.71 65.34	13.223 12.646	-1.576	293	0.116	-2.37

* equal variances assumed (F = 0.277, p = 0.599)

Table 2

T-test for Equality of Mean Pre-Calculus Scores (α = 0.05) by Sensing (S) – Intuition (N) Personality Scale*

Personality	n	Mean	SD	t	df	Sig. (two-tailed)	Mean Difference
Sensing Intuitive	120 175	65.45 67.10	13.787 12.334	1.075	293	0.283	1.65

* equal variances assumed (F = 0.482, p = 0.488)

Table 3

T-test for Equality of Mean Pre-Calculus Scores (α = 0.05) by Thinking (T) – Feeling (F) Personality Scale*

Personality	n	Mean	SD	t	df	Sig. (two-tailed)	Mean Difference
Thinking Feeling	170 125	66.72 66.04	12.429 13.662	-0.446	293	0.656	-0.68

* equal variances assumed (F = 0.168, p = 0.682)

Table 4

T-test for Equality of Mean Pre-Calculus Scores (α = 0.05) by Judging (J) – Perceiving (P) Personality Scale*

Personality	n	Mean	SD	t	df	Sig. (two-tailed)	Mean Difference
Judging Perceiving	188 107	67.60 62.46	12.063 15.020	2.566	92.422	0.012**	5.14

* equal variances not assumed (F = 6.580, p = 0.011), ** $p < 0.05$

As Table 1 shows, the mean Pre-Calculus score for students classified as Introverted was 2.37 points higher than the mean score achieved by students classified as Extraverted. However, at the α = 0.05 confidence level, this difference was not significant (t(293) = -1.576, p = 0.116). Even smaller mean differences were found between groups when the Sensing-Intuition scale and Thinking-Feeling scales were used to divide the sample (see Tables 2 and 3). The mean score achieved by students classified as Intuitive was 1.65 points higher than that achieved by those classified as Sensing, and students classified as Thinking performed only 0.68 points higher than those classified as Feeling. Neither of these differences were significant at α = 0.05 (t(293) = 1.075, p = 0.283; t(293) = -0.446, p = 0.656).

However, a much larger difference was noted when the sample was separated using the Judging-Perceiving scale. Of the 295 students, 188 were classified into one of the eight personality types that included a J (Judging). These students achieved a mean Pre-Calculus score 5.14 points higher than that of the 107 students whose personality types included a P (Perceiving). A t-test for unequal variances found this difference significant (t(92.422) = 2.566, p = 0.012, α = 0.05). Therefore, students classified as Judging scored significantly higher in Pre-Calculus than those classified as Perceiving.

DISCUSSION AND IMPLICATIONS

In this study, the effects of the Introversion (I) – Extraversion (E) scale, the Sensing (S) – Intuition (N) scale, and the Thinking (T) – Feeling (F) scale were not significant on collegiate Pre-Calculus performance. Students on opposing ends of these three scales achieved equivalent levels of success in the course. In contrast, the effect of the Judging (J) – Perceiving (P) personality scale on Pre-Calculus performance was highly significant ($p = 0.012$, $\alpha = 0.05$). When the sample of college students was separated using the antipodal poles of the Judging-Perceiving scale, mean Pre-Calculus scores achieved by the two groups were significantly different. The students classified as Judging scored significantly higher than those classified as Perceiving.

Individuals classified as Judging tend to approach life in a structured, organized manner. They attain a sense of control by planning in advance and organizing the world to achieve their goals. They make quick and clear decisions, are self-disciplined, and may be seen as rigid and opinionated. People classified as Perceiving, on the other hand, find structure limiting and want to keep their options open for new information and possibilities. They are curious, adaptable, and tolerant. They make decisions only when necessary and enjoy the exploration of problems and situations (Straker, 2008). Overall, students classified as Judging focus more on planning and making immediate decisions, whereas students classified as Perceiving are more concerned with studying new information for additional possibilities.

So why are students with Perceiving tendencies performing worse than those with Judging? In the study of mathematics, curiosity and interest in exploration are encouraged and viewed as positive traits of a mathematics learner. Yet, the students in this study who were inquisitive and open to possibilities unexpectedly performed significantly worse than those who showed the very opposite characteristics. From the view of a mathematician, this result may seem counter-intuitive.

On the other hand, this result may not be surprising, considering that the sampled Pre-Calculus classes were mostly taught through lecture. It seems reasonable that a lecture-based teaching format could be more conducive to the learning of students classified as Judging, i.e. those who are more self-disciplined and prefer structure. Unfortunately for students who do not learn well in this type of environment, the majority of mathematics courses at American colleges are taught in a similar fashion. Perhaps mathematics instruction and content in general at the collegiate level is too structured or rigid to support the learning styles of students with Perceiving characteristics.

College mathematics instructors need to be aware of the role personality may play in student achievement, especially when working with students who are not necessarily mathematically advanced. Although educators may be unable to influence the personalities of their students, by understanding their students' personalities, and how they may affect their abilities to learn mathematics, instructors could modify their teaching strategies to address the learning needs of all students. For example, if students with Perceiving characteristics are not profiting from current college mathematics instruction, educators need to explore curricular and instructional changes that could facilitate the learning of these students, while maintaining the success of students with Judging characteristics.

In courses like Pre-Calculus, mathematics instructors need to capitalize on the skills and strengths of students classified as Perceiving. For example, since students classified as Perceiving tend to be curious and investigative, they could benefit from a curriculum focused around problem solving. These students are likely to become bored with rote learning, and problem solving could provide much needed stimulation and engagement. Barger and McCoy (2009) have suggested hands-on, exploratory activities for their collegiate Contemporary Mathematics course. The inclusion of such activities, pertinent to Pre-Calculus, may help address the needs of students classified as Perceiving. Furthermore, these students like to study situations for new information and may therefore benefit from increased visualization in areas such as graphing. The use of a graphing calculator or an online graphing system, such as www.WebGraphing.com, could also be valuable to their learning.

CONCLUDING REMARKS

This study supports previous research in illustrating the importance of certain personality traits in regards to academic achievement (Chamorro-Premuzic & Furnham, 2003; Wagerman & Funder, 2007). Specifically, it reinforces the existence of an important relationship between personality and mathematical success for students with particular personality traits (Karnes, et al., 1984; Siegel & Shaughnessy, 1992; Spray, 1995). However, since little research has addressed the effect of personality on mathematics learning in lower-level collegiate courses, such as Pre-Calculus, this work should be replicated with other entry-level mathematics courses and at additional institutions to validate this finding.

Furthermore, identifying ways in which personality can affect mathematical success is only a first step in addressing the overarching goal of improving college student achievement. Further exploration is needed to identify strategies and formulate recommendations that college mathematics instructors could use to support students, who may not be naturally inclined to act in ways that lead to mathematical success, within the present instructional format of their courses.

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BIOGRAPHICAL SKETCH

Rachael M. Welder is currently an Assistant Professor of Mathematics Education at the City University of New York, Hunter College. She received her Ph.D. in Mathematics, with specialization in Mathematics Education, from Montana State University. In addition, she holds a master's degree in Mathematics from the University of North Dakota. Her research interests include algebra misconceptions and the mathematical preparation of preservice elementary teachers. Dr. Welder has lived in New York City for three years with her husband, Sean, and their two beloved cats, but enjoys traveling back to the Midwest (when the weather cooperates!) to see her family.

Questions, Questions, Questions…Is Questioning an Essential Strategy?

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ABSTRACT

In this article, the author suggests that questioning is an instructional strategy which is commonly ignored in formal lesson planning. The author offers many ideas and suggestions gathered from practitioners for the improvement of this essential instructional skill.

Through my years as a department supervisor, principal, and superintendent my most enjoyable and probably most important task was the coaching of teachers in the classroom. During each visit I would look for the components of a solid lesson: preparation, organization, rapport, time utilization, activities, student engagement, assessment, and closure. In my experience I found that, ironically, the one instructional strategy common to most lessons and regularly ignored in preparation is questioning. This essential strategy became a major focus of my coaching and continues to this day.

Questioning is a possibly potent but often neglected component of the rather amorphous concept referred to as thinking (McKenzie, 2005). In my experience in pre and post conferencing I found that most teachers feel that they are accomplished questioners and therefore take this strategy for granted. Teachers routinely articulate their objectives and carefully craft activities in support of those objectives around major concepts commonly referred to as "essential questions" (Wiggins and McTighe, 2005). They plan for introductory activities, assessment and closure, often utilizing simple questions. Unfortunately, all too often these instructional questions are not what they could be. It is this essential instructional strategy, questioning, that is the focus of this paper.

Why do teachers ask questions?

Cotton (2001) suggests a variety of purposes:

- To develop interest and motivate students to become actively involved in lessons
- To evaluate students' preparation and check on homework or seatwork completion
- To develop critical thinking skills and inquiring attitudes
- To review and summarize previous lessons
- To nurture insights by exposing new relationships
- To assess achievement of instructional goals and objectives
- To stimulate students to pursue knowledge on their own.

Hanson, Silver, and Strong (1986) suggest that every question should be considered part of a larger strategy that includes the teacher's preparation for the question, the question itself, the student's answer and the teacher's response.

I once attended a workshop presented by David R. Johnson which included a segment on questioning which he later included in a publication on instructional strategies (Johnson, 1982). I was so impressed with the practicality of his ideas that as a principal I began to conduct similar activities periodically with my faculty. Now, as a professor of education, I continue to facilitate discussions of questioning with my students. What follows is a compendium of questioning principles originally inspired by Mr. Johnson but expanded by practicing educators over many years.

Whenever possible:

- Know the material well; be confident;
- Prepare questions ahead of time;
- Plan questions for each specific objective in the lesson;
- Plan questions to involve all students and specific students;
- Plan questions to stimulate thought and engagement;
- Anticipate student responses and your subsequent responses.

A "Try to" List:

- Pause after asking a question to allow "wait" time;

- Follow student responses with the question, "why?"
- Ask another student to react to a student's answer;
- Insist on attentiveness and respect;
- Articulate – speak clearly, don't rush;
- Demonstrate appreciation/fascination with the subject matter;
- Invite student questions;
- Ask some open-ended questions;
- Leave an occasional question unanswered (let the conversation continue on the school bus or at home);
- Replace lectures with a set of guided questions;
- Occasionally solicit student questions as "exit tickets" at the end of a lesson to assess their learning and assist your planning;
- Use "communicators" (individual white boards, chalk boards, etc.) for group responses;
- Vary question formats;
- Occasionally build a question around a common misconception;
- Probe but don't badger;
- Make eye contact;
- Ask the students to suggest some common mistakes or misconceptions;
- Ask the students to articulate their thinking (metacognition);
- Ask, "Why do you think I asked this question?"

Try to AVOID:

- Avoid many "yes" or "no" questions;
- Avoid answering your own questions;
- Avoid only "recall" questions;
- Avoid using questions for discipline;
- Avoid repeating student answers;
- Avoid giveaway facial reactions;
- Avoid questions that contain the answer;
- Avoid calling on a student before asking a question;
- Avoid calling on a student immediately after asking a question;
- Avoid labeling the degree of difficulty of a question.

Some "Useless" Questions:

- "How many of you understand that?" (Will they be honest?)
- "Does everybody see that?" (Who should answer for the group?)
- "Do you want me to go over that again?" (No, I didn't want it the first time!)
- "Did I go too fast for you?" (No, let's get to the homework!)
- "This is a right triangle, isn't it?" (Why bother asking?)
- "Right?" or "You know?" (or any other useless habit of speech)
- "Do you have any questions?" (Again, who will be honest?)

Some "Technology" Questions:

- "Where might you find additional information about this topic?"
- "How can you organize and present this information most effectively?"
- "How might we use our laptops, cell phones, etc. to address this problem?"
- "Who can operate this technology?"
- "How might we use networking technology to enhance our cooperation?"
- "Is this answer reasonable?"

Consideration of Bloom's Taxonomy leads us to some "Bloomin' " Questions:

Who can….

- "Define, describe, identify, label,…"
- "Explain, interpret, translate, summarize,…"
- "Apply, solve, construct, compute,…"
- "Analyze, diagram, differentiate, outline,…"
- "Synthesize, summarize, combine, create,…"
- "Evaluate, criticize, compare/contrast,…"

Some "Different" Questioning Ideas:

- Sing, dance or mime a question (or answer);
- Have students write answers on odd surfaces (for texture or visual effect);
- Conduct a "Chain Question" activity such as "I have…who has…?;"
- Ensure 100% participation through the use of name cards, etc.
- Ask a silly question!

When the lesson is over we naturally look ahead and begin to plan for the next day. As an administrator I encourage reflection to facilitate that planning process.

Questions That I Ask Myself after a lesson:

- What worked particularly well in today's lesson? Why?
- What didn't work in today's lesson? Why not? Can I modify it?
- Should I present this material some other way next time? How?

Max Sobel, Professor Emeritus of Mathematical Education at Montclair State University, once concluded a professional workshop for teachers with a list of questions for professional self-examination. I quote him at the end of my workshops:

- Has each pupil profited by my presence in the classroom today?
- Did I demonstrate my love for the subject?
- Did I show that I also love to TEACH this subject?
- Did I meet the needs of each of my students?
- Did I teach with patience and humor?
- Did I encourage my students to do their best?
- Did I display the enthusiasm that is so contagious and so essential for success in the classroom?
- Did I make each of my students feel important today?

Periodically, I step back and examine my learning environment by asking students to grade ME as a teacher. I peruse their comments and evaluations for evidence of effectiveness in areas suggested by the Mathematical Sciences Education Board National Research Council (1991).

- Do I encourage students to explore?
- Do I help students to verbalize their mathematical ideas?
- Do I show students that there is often more than one correct answer?
- Do I teach my students, through experience, the importance of careful reasoning and disciplined understanding?
- Do I provide evidence that mathematics is alive and exciting?
- Do I build confidence in all my students that they can learn mathematics?

I want to close with a thought from Socrates, "Wisdom begins in wonder." Let's do all we can to ignite that wonder in our students.

Finally, I want to express a sincere "Thank you" to all those wonderful teachers who have shared their questioning suggestions with me and others over the years!

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