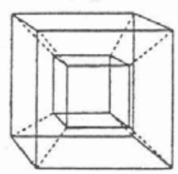

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Editorial

Curriculum Focal Points in New Jersey

Late last year The National Council of Teachers of Mathematics (NCTM) published a new document, Curriculum Focal Points for Prekindergarten through Grade 8 Mathematics, suggesting curricular reform. The release of this document sparked some media reporting as it had been presented to the National Math Panel, the Conference Board of the Mathematical Sciences, the American Mathematical Society's Committee on Education, the Joint Mathematics Meetings of the Mathematical Association of America and the American Mathematical Society, regional meetings of the U.S. Department of Education's Mathematics and Science Partnership projects, and House and Senate staff members. This document presents three major mathematical concepts, skills or understanding for each grade level. The purpose of the focal points is "to ensure direct attention, or focus, so that what is taught can be covered thoroughly and understood deeply, with continuous engagement in problem solving, reasoning and proof, communication, connections, and related representations", according to the NCTM website (http://www.nctm.org/about/president.aspx?id=6310&blogid=68).

How this document will affect teaching and learning in New Jersey is still not clear. It is hoped that careful and reflective analysis will lead to mathematical leaders at the state and local school level to determine the important mathematics at each grade level and so provide teachers with clear direction for future curriculum decisions.

The current issue focuses on a variety of topics, and levels of mathematics, that would appeal to both teachers and teachers of teachers. Karen Heinz highlights the use of word problems to promote understanding of how to divide by a fraction, while Patricia C. Kenschaft addresses the mathematical preparation of elementary school teachers. Jay L. Schiffman presents a number of diverse settings focusing on a geometric progression (2^n-1). Jane Wilburne focuses on the misrepresentation of an egg timer and Robert Mitchell investigates 495's surprising property. Enjoy!

Suriza van der Sandt, Editor, AMTNJ Journal

Preparing Elementary School Teachers Mathematically

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In the fall of 2006 I fulfilled a 15-year-old dream. I taught a class consisting primarily of college students hoping to become elementary school teachers a mathematics course designed especially for their needs. It was wonderful. They learned, and we all had fun – most of the time! I will teach it better next time, but there is much to share already about what we did and why it is important.

At first they resisted doing mental math, but they eventually reveled in it. The discussions were revealing. If you want to *subtrac*t 99 from 174, then adding one to both numbers makes finding the difference easy. One student asked, "When you are *adding* 99 to 174, why don't you add one to both sides, as we do in subtraction?" Dependence on rules like, "Do the same thing to both sides of the equation," is, alas, rampant – even if there is no equation! One remedy to this misunderstanding is a plethora of pictures that show addition as putting two bars end to end and subtraction as finding the difference between them.

Other misused rules are harder to unravel. How do you respond to a "rule" that *all* improper fractions *must* be converted to mixed numbers? What does this rule do to the process of multiplying fractions? It took considerable persuasiveness to convince them that improper fractions, however pejorative their name, have a legitimate role in this world.

Some rules were more difficult for me to decipher in context. The text suggested multiplying 9 x 89 mentally by converting 9 to 10 – 1. Many of the students could then compute 890 – 89 in their heads, but some didn't "get" the process. I wrote on the board

$$9 \times 89 = (10 - 1)89$$

and for a while I simply couldn't understand why one persistent student kept saying, "order of operations!" Over night I realized she had been indoctrinated in the practice of always doing the process in the parentheses first, and she thought I was violating an immutable mathematical rule by applying the distributive law before doing the subtraction indicated in the parentheses. The following day I apologized for not hearing what she was saying, and explained that sometimes we could reverse the order of operations if it served our purposes. She smiled wryly over the whimsies of authority figures, and now followed my explanation without trouble.

This takes us to the topic of the distributive law. They know "FOIL" well, but are pretty shaky on its justification. I remember how impressed I was when Mary Dolciani introduced FOIL in her texts, a few years after I had had to struggle directly with the distributive law in high school, but now I can see its downside. I believe my students have become enlightened about the confusions explained in the previous three paragraphs, but I'm not so sure about the distributive law. "Why not just use FOIL?" they would say when appropriate, and in other contexts, they seemed to have trouble recognizing and using the distributive law.

More controversial than all this was the use of the equals sign. I've been annoyed for years by students, even well up in the calculus sequence, who use "=" to mean "and then you do" instead of "is the same as." I would make changes on papers, sigh, and move on. But now I was teaching people who would have an impact on the next generation of learners! I believe "=" should mean only "is the same as." This may be parochial of me, but it has served many generations well. It's an established notation in mathematics. What would we do if the traditional equals sign were no longer available?

I kept mentioning this traditional belief (and custom), and kept feeling I was being ignored. We all have times when we feel ignored, and I wasn't sure what to do about it. The next to last quiz, two weeks before the end of the semester, I decided to dock students a point for each time they misused "=." Handing back those papers resulted in the time when we had the least fun of the semester. They were indignant. How dare I take a point off for *that*? I felt that my message was noticed now, but I wasn't necessarily victorious in winning converts and influencing people, as much as I tried to defend the old customs about "=." They forgave me by the next class, but I'm not sure I conveyed the crucial nature of "equals."

Even more pernicious than misguided applications of rules are key words. The worst use of key words in my teaching career is given below, but this semester's worst was probably when I was trying to convey the two interpretations of division. (Partitive: "6 ÷ 2" means, "If you divide six into two parts, how many are in each part.?" Gazinta: "6 ÷ 2" means, "How many times does two go into (gazinta) six?" or "How many twos are there in six?") One of my best students said she knew a problem involved partitive division if it included the phrase, "How many?" When I said that key words were not a satisfactory way to understand a mathematical concept, another student became quite belligerent. "Well, how are you supposed to tell?" It seemed that she had little concept of understanding mathematics as other than a word game. By the end of the semester, I think the students realized that learning mathematics *can* be something other than memorization of key words and rules. If so, that was my greatest achievement with them.

How do people learn mathematics? "There is no research on how people learn division," said Skip Fennell, president of the NCTM, to a group of us gathered after a session with him and the chair of the National Mathematics Panel on January 8, 2007, in New Orleans. The Panel has not discovered any psychological research on how people learn division. He emphasized this is in great contrast to the enormous amount of research on how children learn more elementary mathematics. When I suggested this might be because the researchers don't understand division and fractions, nobody challenged me.

I was already concerned about the teaching of fractions in New Jersey schools fifteen years ago when I first went into a fifth grade class in one of New Jersey's richest public school systems. Friendly faces smiled up at me, eager to please and expecting to be pleased.

"Where is one third on the number line?" I began.

The friendly eyes all fell to the floor, staring at some unseen attraction down there. I repeated the question.

"It's near three, isn't it?" suggested the teacher, one of the most highly paid fifth grade teachers in New Jersey.

A friend observing another fifth grade class in another suburb watched the teacher tell the children to add fractions by adding first the numerators and then the denominators. Another friend saw a high school teacher repeatedly say things like, "Five divided by fifteen is three." College students often reverse the order of division, and some seem to think division is commutative.

No wonder so many American adults have trouble understanding the basic mathematics that underlies our far-reaching economic and environmental decisions! Elementary school teachers are part of this public, but they are also setting the mathematical habits that affect how future and current citizens use patterns in our civic life. If youngsters are taught wrong mathematics and also the lesson that smart leaders (like their teachers) can't do mathematics, it is very difficult for the mathematically knowledgeable teachers that the lucky ones meet in secondary school to undo the damage.

Are elementary school teachers capable of learning division and fractions? Yes! Why, then, are they and such a large portion of the American public baffled by these concepts? It is a vicious cycle. If they haven't been taught, they are not prepared to teach. "You can't teach what you don't know, any more than you can come back from where you ain't been," observed Will Rogers long ago.

It doesn't have to be so. I remain convinced that elementary school teachers are capable of learning division and fractions. I used the text *Elementary Mathematics for Teachers* by Thomas H. Parker and Scott J. Baldridge, which is based on five elementary school texts from Singapore.[1] The six paperback books cost my students $67. Singapore often scores first on international tests, and their texts are in English. They are 7.5" x 10.25" paperbacks less than a quarter inch thick for each half-year. Fall semesters are arithmetic and spring provide geometry; our books were all from the arithmetic sequence.

They include everything the students need to know with plenty of exercises but no padding. The pictures illustrate only the mathematics. There is widespread use of "bar diagrams" whose lengths represent numbers. These are used to depict ratios so "visual learners" (and the rest of us) have a tangible way to perceive abstract concepts. Bar diagrams are used with addition and subtraction (as mentioned above), so the students are ready to use them to show fractional amounts. My students told me at the end of the semester that they were sharing bar diagrams with children and peers with great success.

The Singapore texts also use rectangles abundantly to illustrate fractions. To add, for example, 1/3 + 2/5, one rectangle is cut horizontally in thirds and a congruent rectangle is cut vertically in fifths, and the indicated quantity is shaded. On the next line the first is cut vertically in fifths and the second horizontally in thirds. The student can now see that both shaded areas can be represented by fifteenths, and they can see why you multiply to get equivalent fractions with a common denominator. My students and I found this exciting. Studying fractions only as pieces of pies is suffocating. Rectangles, bars, and number lines are also needed *early* in a child's adventures with fractions.

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[1] Thomas H. Parker and Scott J. Baldridge, *Elementary Mathematics for Teachers,* Sefton-Ash Publishing, Oregon City, OR, 2004, available at www. singaporemath.com

A national survey publicized toward the end of the semester indicated that only 20% of adult Americans can compute their own gas mileage.[2] I asked my students if any of them could. None admitted to being able to do so. We were about to study rates, so we spent considerable time computing gas mileage, sometimes using data that they and I brought in from our own cars and gasoline purchases. They could do it themselves by the next quiz.

They could even share my shock at one of the incidents that caused me to pursue teaching prospective elementary school teachers vigorously. On the last day of a liberal arts course, one that fulfills the entire mathematics requirement for prospective elementary school teachers, a nice pre-service elementary school teacher said there must be something wrong with one of the problems in the text. "The fuel mileage of light trucks owned by U.S. families in 1999 was 20 mpg and of sedans was 28 mpg. Altogether they were 23 mpg. But 'altogether' means 'add,' so altogether they must have a fuel mileage of 48 miles per gallon." I explained why the mileage altogether must be between 20 and 28 mpg three times, which is my limit. Then I asked for help. Three students gave what I thought were fine explanations, but the future elementary school teacher clearly didn't understand. She was polite and she knew she was outvoted, but how could a rule she had been so frequently taught be wrong?

"You know, Dr. Kenschaft, key words," said one of her classmates.

"And left always means subtract," chimed in another. (Key words dominate the commercial programs that districts are urged to buy to prepare children for standardized tests, but that is another article.)

A faculty member at a New Jersey teacher preparation institution where I have never taught told me of reluctantly giving a D- to a student who had postponed her only math course to the last semester of her senior year so that the student could graduate. The following year the graduate told her former math professor that she was now teaching eighth grade mathematics full-time. "So I guess she's ruining 125 students a year, and it's all my fault," mused my friend.

From 1988 to 1995 I won 14 grants to help current elementary school teachers mathematically and taught hundreds of elementary school mathematics classes.[3] My impression of the teachers with whom I worked in nine school districts, both urban and suburban, was that the difference between the urban and suburban schools' mathematical achievement is because some of the suburban parents teach their children mathematics at home, and these children teach their peers.[4] The teachers seemed to me to be of similar quality. All were aware of their severe mathematical shortcomings. The ones who joined

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[2] National Center for Education Statistics, "National Assessment of Adult Literacy, a First Look at the Literacy of America's Adults in the 21st Century," NCES publication number 2006470, Dec. 2005, available at http: //nces.ed.gov/naal/

[3] For an account of my adventures during that time, see "Racial Equality Requires Teaching Elementary School Teachers More Mathematics," in the *Notices of the American Mathematical Society*, February 2005, 208-212, available at www. ams.org/notices/200502/fea-kenschaft.pdf

[4] Convinced that currently parents are our only hope, I wrote *Math Power: How to Help Your Child Love Math Even If You Don't* for parents of children aged one through ten. Republished by Pi Press, NYC, 2006.

our program were eager and quick to learn. Neither motivation nor ability was a problem.

One memorable day I taught a group of teachers how to illustrate 14 x 22 using base ten blocks, and then tied the visual message to the standard multiplication algorithm for two-digit numbers. "Why wasn't I taught this secret before?" exclaimed one of the teachers. She demanded whether each of the others had known "this secret" and established that it had been kept from them too. Then she turned on me, deeply angry. "Why wasn't I taught this secret before? I've been teaching third grade for thirty years, and I could have been a *much* better teacher if someone had let me in on this secret thirty years ago!" Her fury continues to fire me. I have taught the area of a rectangle to hundreds of third graders, apparently without failure (although one can't be sure about that).

New Jersey should require that all prospective elementary school teachers take mathematics courses designed especially for them *before* they damage their pupils. The No Child Left Behind act requires a "highly qualified teacher" to be in every classroom, but defines "highly qualified teacher" as one who has a bachelor's degree and state certification. This is inadequate if state certification does not require subject matter knowledge. The National Council of Teachers of Mathematics, in a 2005 statement, asserts that a "highly qualified teacher" is "a teacher who knows mathematics well and who can guide students' understanding and learning… Elementary teachers... should have completed the equivalent of at least three college-level mathematics courses that emphasize the mathematical structures essential to the elementary grades (including number and operations, algebra, geometry, data analysis, and probability.)"[5]

This 2005 NCTM statement reflects the recommendations in the book, *The Mathematical Education of Teachers*, released by the Conference Board of the Mathematical Sciences (CBMS), a group of sixteen mathematical organizations, in 2000. It calls for improved teacher preparation at all levels. It claims, as I have discovered, that "teaching elementary school mathematics can be intellectually challenging" in an appeal for members of the Mathematical Association of America and the American Mathematical Society to undertake teaching prospective and current elementary school teachers the mathematics that they should be teaching children.[6] This document describes in detail the content of four areas of elementary school mathematics: number and operations; geometry and measurement; data analysis, statistics and probability; and algebra and functions.[7] Louisiana[8] and Oklahoma[9] require four college courses in mathematics for pre-service elementary school teachers, and most institutions provide special courses for them. Georgia[10] requires four courses designed for prospective

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[5] National Council of Teachers of Mathematics (NCTM), "Position Statement on Highly Qualified Teachers," July 2005. www. nctm.org/about/position_statements/qualified.htm

[6] CBMS, *The Mathematical Education of Teachers*, American Mathematical Society in cooperation with the Mathematical Association of America, Providence, RI, 2000, p. 15. available at www. cbmsweb.org.

[7] Ibid, p. 18-23.

[8] personal email from Judith Covington, Louisiana State University at Shreveport, Feburary 1, 2007.

[9] personal emails from David Boliver, University of Oklahoma, February 9, 2006 and January 30, 2007.

[10] personal email from David Stone, Georgia Southern University, July 13, 2006.

elementary school teachers, but allows them to substitute a course in "concepts of calculus" for one of the nationally recommended courses.

Reasons to introduce such courses gradually include (1) political reality and (2) the availability of good teachers to teach future teachers, but New Jersey should start such a program soon. Wisconsin[11] currently requires three of these courses. North Carolina requires two[12], although some institutions do not have special courses for elementary school teachers.

In 2004 Conference Board of the Mathematical Sciences released another document saying, "The need for sound mathematical preparation of teachers is acute."[13] Providing teachers opportunities to learn the mathematics they are supposed to teach will do much more to help children (and teachers) than the far more expensive testing programs that now possess our state and nation. As a blue ribbon national panel led by Governor James B. Hunt of North Carolina concluded in 1989, "…increased graduation and testing requirements only create greater failure if teachers do not know how to reach students so that they can learn."[14]

This report asserts, "Literally hundreds of studies confirm that the best teachers know their subject matter deeply…"[15] One such study in New York City that compared high-achieving and low-achieving schools with similar students concluded that teacher qualifications explained 90% of the difference.[16] Another in Texas found that after controlling for socio-economic status, the large disparity between achievement levels of black and white students were almost entirely explained by differences in the qualifications of their teachers.[17]

One course clearly doesn't do the job, but it is a start. My first effort, reported in the first half of this article, was a specially registered class within the "Mathematics for Liberal Arts" course. My joy with this experience resulted in Bloomfield College's implementing a new course "Mathematics for Elementary School Teachers" into its program, which prospective elementary education teachers will take instead of "Mathematics for Liberal Arts." I will be teaching this for the foreseeable future.

At the end of the semester, my students complained they weren't prepared to teach fourth grade arithmetic yet. They to learn need more. "Yes," I replied, "but if you must teach fourth or fifth grade instead of the primary grades that you want to teach, you will be better prepared than most New Jersey fourth and fifth grade teachers. And because you know you don't know, you will keep learning mathematics." They nodded sadly. (I

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[11] personal email from Richard Askey, University of Wisconsin, June 30, 2006.

[12] personal email from Lee Stiff, North Carolina State University, June 30, 2006.

[13] www. maa.org/cupm/curr_guide.html, p. 39.

[14] National Commission on Teaching & America's Future, *What Matters Most: Teaching for America's Future*, National Commission on Teaching & America's Future, NYC, 1996, p. 5.

[15] Ibid, p. 52.

[16] Eleanor Armour-Thomas, Camille Clay, Raymond Domanico, K. Bruno, and Barbara Allen, *An Outlier Study of Elementary and Middle Schools in New York City: Final Report,* New York City Board of Education, 1989.

[17] National Commission on Teaching & America's Future, *Doing What Matters Most: Investing in Quality Teaching*, National Commission on Teaching & America's Future, NYC, 1997, p. 8, quoting Ronald Ferguson, "Paying for Public Education, How and Why Money Matters, *Harvard Journal of Legislation*, 28, Summer 1991, pp. 465-98.

had told them of the surveys indicating there are many more jobs available in fourth and fifth grade than the primary grades.) “And now you know you *can* learn mathematics, so you will work at it.” This time they smiled, and nodded more vigorously.

References:

Advisory Committee on Improving Teacher Content Knowledge in Mathematics, for the United States Department of Education, R. James Milgram, principal author and editor, *The Mathematics Pre-Service Teachers Need to Know*, (580 pages), 2005. ftp://math.stanford.edu/pub/papers/milgram/FIE-book-high-res.pdf.

Armour-Thomas, Eleanor, Camille Clay, Raymond Domanico, K. Bruno, and Barbara Allen, *An Outlier Study of Elementary and Middle Schools in New York City: Final Report*, New York City Board of Education, 1989.

Conference Board of the Mathematics Sciences (CBMS), *The Mathematical Education of Teachers*, American Mathematical Society in cooperation with the Mathematical Association of America, Providence, RI, 2000. available at www. cbmsweb.org.

Conference Board of the Mathematics Sciences (CBMS), www. maa.org/cupm/curr_guide.html, p. 39.

Ferguson, Ronald, “Paying for Public Education, How and Why Money Matters, *Harvard Journal of Legislation*, 28, Summer 1991.

Kenschaft, Patricia C., *Math Power: How to Help Your Child Love Math Even If You Don’t*, republished by Pi Press, NYC, 2006.

Kenschaft, Patricia C., “Racial Equality Requires Teaching Elementary School Teachers More Mathematics,” in the *Notices of the American Mathematical Society*, February 2005, 208-212, available at www. ams.org/notices/200502/fea-kenschaft.pdf

National Center for Education Statistics, “National Assessment of Adult Literacy, a First Look at the Literacy of America’s Adults in the 21st Century,” NCES publication number 2006470, Dec. 2005, available at http: //nces.ed.gov/naal/

National Commission on Teaching & America’s Future, *Doing What Matters Most: Investing in Quality Teaching*, National Commission on Teaching & America’s Future, NYC, 1997.

National Council of Teachers of Mathematics (NCTM), “Position Statement on Highly Qualified Teachers,” July 2005. www. nctm.org/about/position_statements/qualified.htm

Parker, Thomas H. and Scott J. Baldridge, *Elementary Mathematics for Teachers,* Sefton-Ash Publishing, Oregon City, OR, 2004, available at www. singaporemath.com

Using Word Problems to Promote Understanding of How to Divide by a Fraction

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Division of fractions is notorious for being a challenging topic for students. Also, it is commonly known that many students do not like word problems and that they typically find them difficult to solve. Thus, we may think that we should not ask students to solve word problems involving division of fractions until they have mastered the skill of dividing fractions. However, as I discuss in this article, an appropriate set of word problems can actually be a fruitful place to begin an exploration of division of fractions. The goal is for students to learn how to divide by fractions in a way that makes sense rather than just memorizing the invert-and-multiply algorithm (i.e., to determine the answer to $6 \div \frac{2}{3}$ one would multiply 6 by $\frac{3}{2}$).

In the word problems I present in this article, people at a party share a given number of pizzas. Each person receives the same fractional amount of pizza. In mathematical terms, we can say that the pizza is being put into groups of equal size. When thinking of division in terms of equal-sized groups, there are two ways to interpret the divisor and the quotient. In one interpretation, referred to as the partitive or sharing model of division, the divisor indicates the number of groups and the quotient is the number (or amount) in each group. In the other interpretation, known as the quotative or measurement model, the divisor indicates the number (or amount) in each group, and the quotient is the number of groups. Students typically find it easier to think about dividing by a fraction if the fraction indicates the amount in each group, so the word problems that I present all fit the quotative model of division.

Finding the Total Number of Pieces: Learning the Role of Multiplication in the Algorithm

An example of an initial word problem to pose to students is the following. "Six pizzas were delivered to people at a party. After each person took $\frac{1}{2}$ of a pizza, there was no pizza left. How many people were at that party?" Note that students do not need to know how to perform a division algorithm to solve this problem. If students do not know of a more sophisticated way to solve this problem, it is likely that many will still be successful if they make a drawing, as shown in figure 1. Students can draw 6 circles to represent the 6 pizzas and divide each circle in half. Then, all they have to do is count the number of halves to determine that there were 12 people at the party.

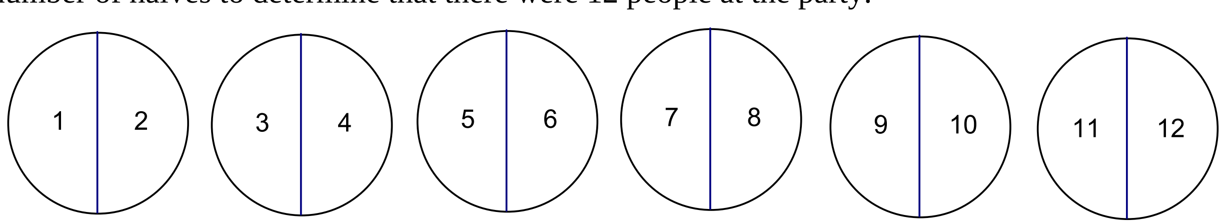


Figure 1: Using drawings to find the number of halves in 6 wholes.

As students work on a set of problems of this type, they begin predicting what the answer is going to be, which is exactly what we want to happen. The other problems in such a set could also be about a pizza party, but contain different numbers. Some possibilities include finding the number of people at the party if (a) 2 pizzas are delivered and each person takes $\frac{1}{3}$ of a pizza, (b) 5 pizzas are delivered and each person takes $\frac{1}{4}$ of a pizza, (c) 3 pizzas are delivered and each person takes $\frac{1}{5}$ of a pizza, and (d) 4 pizzas are delivered and each person takes $\frac{1}{5}$ of a pizza.

After working on a varying number of these types of problems, the students can begin to generalize their procedure. That is, they can learn to solve the problem without drawing the picture. For example, for the last problem stated above students might reason: "Since each person ate $\frac{1}{5}$ of a pizza, I know that the pizzas were cut into 5 pieces. So, each pizza feeds 5 people. Since there were 4 pizzas, I know that there must have been 4 times 5 or 20 people at the party." At this point, students have learned to solve the problem by multiplying the denominator by the whole number, which is an important step in having them understand the invert-and-multiply rule for dividing by a fraction.

Writing an Equation: Learning That the Problem is Modeled by Division

The reader should note that students can solve the problems above without realizing that the problems can be modeled with division, and that is fine. It is often useful for students to solve problems using a less sophisticated method before learning which operation can be used to model the situation. In this case, by solving the word problems by making drawings, students are learning what they are starting with, what is being done to that amount, and what they need to do to find the answer. Students must understand these aspects of the problem in order to be able to learn in a meaningful way that these types of problems involve dividing by a fraction.

If students are not proficient in recognizing division situations in a context and in providing justifications for why a situation involves division, then it might be helpful first to revisit these ideas in a context that involves whole numbers. For example, students could be asked to solve the problem: "A clown has 12 balloons. If she gave away all of her balloons by giving 2 balloons to each child, to how many children did she give balloons?" This simple problem can be used as a focal point for a discussion in which students list the characteristics of a division problem.

When helping students learn which operation to use and why, some key questions to have them focus on are, "What are we given in the problem?", "What is the action involved?" and "What are we being asked to find?" Students should be encouraged to answer these questions as they relate to the specific problem and in more general terms. Sample answers with respect to the balloon problem are the following. We are told that someone has 12 balloons, and the action is that 2 balloons are given to each child. We are being asked to find the number of children who received balloons. In general terms, we can say that we are given a set of objects, and we are to put them into groups of 2 each. We are being asked to find how many groups of 2 can be made from 12 objects.

Then, if students do not state this independently based on their prior knowledge, the teacher can explain that these key components of the problem are what make it a division problem. The summary that the teacher can provide is that since there is a certain number of objects that are being put into equal-sized groups, we can find the number of groups by dividing the total number of objects by the number of objects in each group.

An understanding of these key components of this type of division situation can be useful to students as they work on identifying which operation can be used to model the pizza party situations. After students have solved the previously stated set of problems involving division by unit fractions (fractions with a numerator of 1), they can be asked to write equations (e.g., $6 \div \frac{1}{2} =$) that can be used to solve the problems. This will require them to choose an operation. If students need support in completing this task, the teacher can focus them on the same three questions written above: What information are we given? What is the action involved? and What are we being asked to find? Again, it is useful if students can answer these questions in specific and in general terms. The general answers to these questions are that we are given an amount of pizza; the action involves putting the same amount into each group; and we are being asked to find the number of groups. This means that we can find the number of groups by dividing the total amount by the amount in each group.

Thus far I have presented and discussed activities that can be used to help students learn how to find answers to problems involving dividing by a fraction and how to recognize that these are division problems. In the next section I focus on how word problems can be used to help students learn how to divide by fractions that are not unit fractions.

Sharing a Larger Number of Pieces: Learning the Role of Division in the Algorithm

I have found it useful to ask students to work on problems that have related parts. In the first part, the divisor is a unit fraction. In the second part, the divisor is not a unit fraction but it has the same denominator as the unit fraction in the first part. For example: "Six pizzas were delivered to the people at a party. (a) After each person took $\frac{1}{3}$ of a pizza, there were no pizzas left. How many people were at that party? (b) After each person took $\frac{2}{3}$ of a pizza, no pizzas were left. How many people were at that party?"

When solving this problem, students can use various methods, and they can always rely on drawing pictures to find the answer. In fact, I encourage students to draw a picture for both parts, because it seems to help them develop an understanding of the relationship between the two parts. The drawings that students might make to solve parts (a) and (b) of the above problem are shown in figures 2 and 3, respectively. In figure 2, each person received $\frac{1}{3}$ of a pizza, so there must have been 18 people at the party. In figure 3, since each person received $\frac{2}{3}$ of a pizza, there was only enough pizza for 9 people.

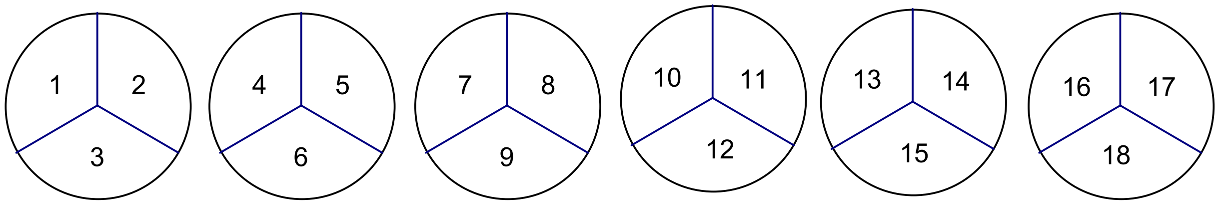


Figure 2: Using drawings to determine that $6 \div \frac{1}{3}$ is 18.

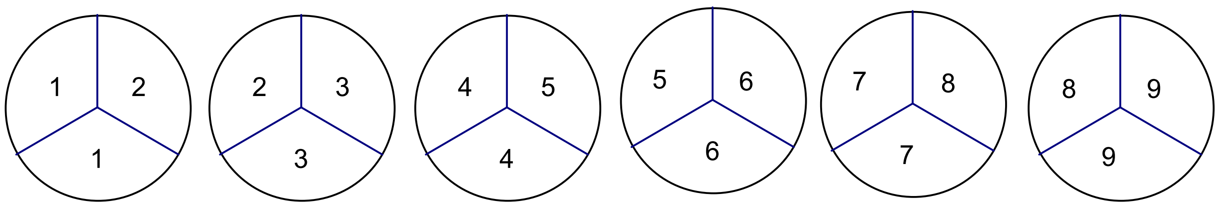


Figure 3: Using drawings to determine that $6 \div \frac{2}{3}$ is 9.

Some students may generalize a procedure after solving one problem, but typically students need to work on multiple sets of problems. Some useful problems are ones that involve the same pizza party situation, with the following variations:

1. 6 pizzas where each person took $\frac{1}{4}$ in part (a) and $\frac{3}{4}$ in part (b)
2. 6 pizzas where each person took $\frac{1}{5}$ in part (a) and $\frac{3}{5}$ in part (b)
3. 5 pizzas where each person took $\frac{1}{6}$ in part (a) and $\frac{5}{6}$ in part (b)
4. 4 pizzas where each person took $\frac{1}{5}$ in part (a) and $\frac{2}{5}$ in part (b) and $\frac{4}{5}$ in part (c)

After working on these types of problems, students may independently start to recognize the key relationships involved and start to solve the problems without drawing the pictures. If not, the teacher can encourage students to reflect on their actions by giving them another problem and asking them to solve it without making a drawing, though if needed, they may mentally picture the drawing. After some time, a thought process that students may be able to engage in for problem #4 above is,

> "If there are 4 pizzas and if each person ate $\frac{1}{5}$ of a pizza, then there must have been 20 people at the party. The reason is that there were five one-fifths in each whole, and there were 4 whole pizzas, so I can find the total by multiplying 4 times 5. In the second part, the fact that each person ate $\frac{2}{5}$ of a pizza means that each person will be getting twice as much. Therefore, there must have been only

half as many people at the party. So, the answer to $4 \div \frac{2}{5}$ is half of 20, which is 10."

A similar line of reasoning can be applied to part (c) to explain why the answer to $4 \div \frac{4}{5}$ is one-fourth of 20. The drawings that students might make to support their explanations for parts (a), (b), and (c) are shown in figures 4, 5, and 6, respectively.

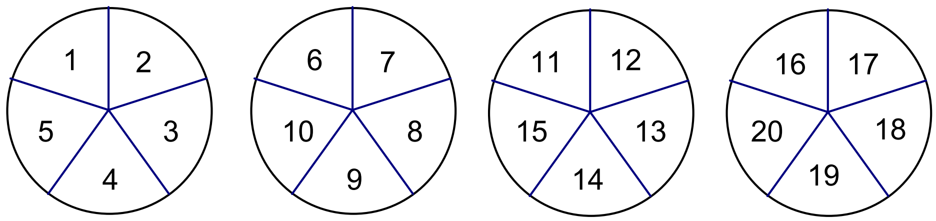


Figure 4: Using drawings to determine that $4 \div \frac{1}{5}$ is 20.

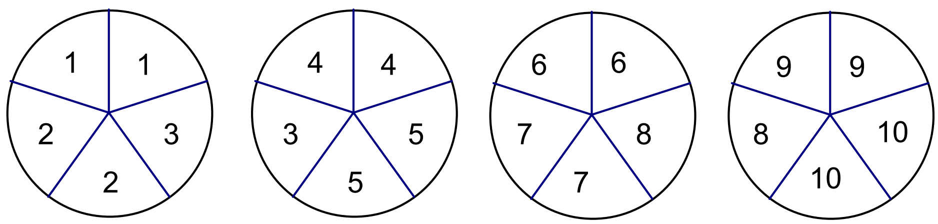


Figure 5: Using drawings to determine that $4 \div \frac{2}{5}$ is 10.

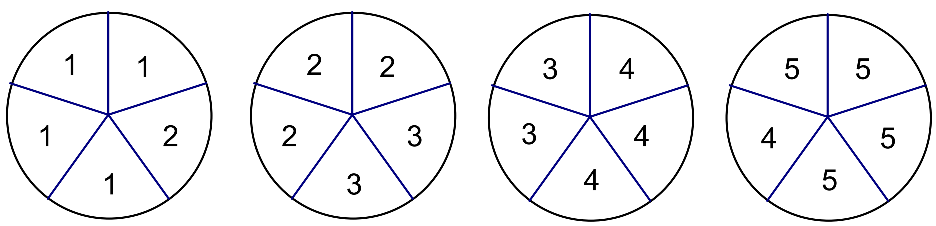


Figure 6: Using drawings to determine that $4 \div \frac{4}{5}$ is 5.

Concluding Remarks

The activities described in this article are only a start. There are still many aspects to division of fractions that students need to explore (e.g., situations where the dividend is not a whole number). However, by working on the types of problems discussed in this article, students have the opportunity to learn that they can solve problems such as $6 \div \frac{2}{3}$ by first multiplying by 3 and then dividing by 2, and to justify *why* that is a valid procedure. In other words, students have the opportunity to re-invent the invert-and-multiply division algorithm and understand why it is a meaningful way to find the answer when dividing by a fraction.

The Misrepresentation of an Egg Timer

Jane Murphy Wilburne
Penn State Harrisburg

The Principles and Standards for School Mathematics (NCTM, 2000) states "problem solving means engaging in a task for which the solution method is not known in advance" (p. 52). The process of problem solving engages students in mathematical investigations and higher order thinking activities and should be an integral part of all mathematics learning. A good problem is one that can be solved using a variety of strategies and can integrate significant mathematics. And many interesting problems can be framed around typical everyday experiences that can lead to the development of mathematical ideas.

To solve a problem, students need to understand the problem and the given information (Polya 1957). However, teachers must remember that students may be exposed to different interpretations of a situation or have prior experiences that affect their understanding of a problem. Consider the following problem, which was posed to a group of middle school students:

> Using two egg timers, one of which runs for exactly 7 minutes and the other for exactly 11 minutes, show how to time the cooking of an egg for exactly 15 minutes. (Posamentier and Krulik 1998, p. 26).

The students began solving the problem using strategies such as making tables and drawing pictures to solve the problem. As the teacher walked around to facilitate the class, he heard students making comments like, "just set the timer to 15 minutes" and "turn the dial to 15 minutes and then when it rings, the egg is done." The teacher began questioning them to hear their explanations. To his surprise, this problem (which was so clear to him) was conveying a completely different message to his students. As the students explained their thinking and drew pictures of an egg timer, it became clear that their representations of an egg timer were totally different from the type he interpreted in the problem (see Figure 1).

Figure 1: Students' Interpretation of an Egg Timer

Digital egg timer

Since the problem assumes the use of a sand timer with no markings (see fig. 2), the teacher determined the problem needed further explanation and even a visual clue of what an "old fashioned" egg timer looks like. Even Webster's dictionary (1994) defines an egg timer as a small sandglass for timing the boiling of eggs.

Figure 2: Sand Timer

Connecting the Problem Solving and Representation Process Standards

The representation process standard (NCTM, 2000) promotes the use of a visual interpretation or graphic when solving mathematical problems to help students solve mathematical problems. In this example, we see the value of using a picture to ensure the meaning of a problem is clearly conveyed and the students use the appropriate interpretation of an egg timer.

Conclusion

Choosing worthwhile problems and mathematical tasks is crucial in teaching. By anticipating the mathematical interpretations and representations exhibited by the students, teachers can modify problems to be sure the wording and terminology is appropriate and understandable Perhaps the egg timer problem would be better suited using buckets of water that hold 7 gallons and 11 gallons to make 15 gallons.

For the reader interested in a solution to the sand egg timer problem: Start the two egg timers together. Do not begin to cook the egg yet. When the 7-minute timer is done, there are four minutes left on the 11-minute timer. Begin to cook the egg now. When the 11- minute timer is done, flip it over. Continue cooking the egg until the 11-minute timer is done.

For the reader interested in a solution using a digital egg timer: Just set the timer for 15 minutes!

Bibliography

Merriam-Webster's Collegiate Dictionary, Tenth Edition. Springfield, MA: Merriam-Webster,

Incorporated, 1994.

National Council of Teachers of Mathematics (NCTM). *Principles and Standards for School*

Mathematics. Reston, VA: NCTM, 2000.

Polya, George. *How to Solve It*. Princeton, NJ: Princeton University Press, 1957.

Posamentier, Alfred, & Stephen Krulik. *Problem-Solving Strategies for Efficient and Elegant*

Solutions: A Resource Book for the Mathematics Teacher. Thousand Oaks, CA: Corwin Press, Inc., 1998

EXPLORING SEVERAL RICH PROBLEM SOLVING ACTIVITIES WITH THE SUM OF A FINITE GEOMETRIC PROGRESSION

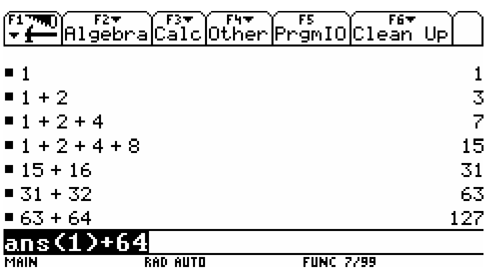
Jay L. Schiffman
Rowan University, NJ

Introduction

The NCTM at its Annual and Regional Conferences often devotes some of its sessions to delving deeply into a single problem. This is often classified as one rich problem session. To this end, consider the geometric progression $1+2+4+...+2^{n-1}=2^n-1$ occurring in a number of diverse settings in elementary mathematics. The purpose of this article is to explore this progression and discuss several rich problem-solving activities encompassing number bases, elementary number theory, and discrete mathematics. These diverse activities revolve around this single idea.

<u>Activity 1: A Popular Problem:</u>

The problem describes a child who possesses mathematical acumen and unsuspecting parents. Instead of accepting a lump sum allowance of $100 over the calendar month, the child wishes to have the allowance apportioned as follows: Pay one penny the first day, two cents the second day, four cents the third day, eight cents the fourth day, etc. (the allowance is doubled each day.) What is the total allowance the child will receive at the end of a thirty-day calendar month? The class decided to split the calendar month into three ten-day segments. With the aid of their calculators, the following data representing the cumulative total after a given number of days was displayed in FIGURES 1-5 below:



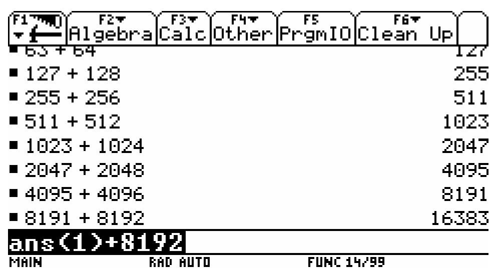
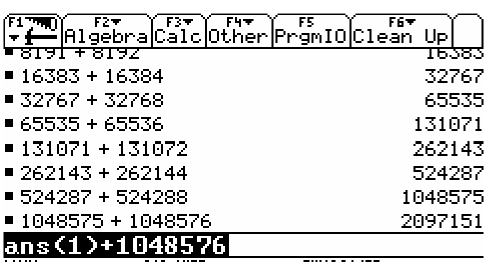


FIGURE 1 FIGURE 2



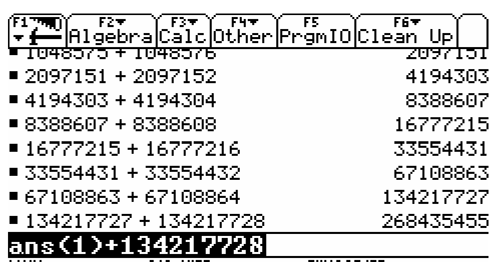


FIGURE 3 FIGURE 4

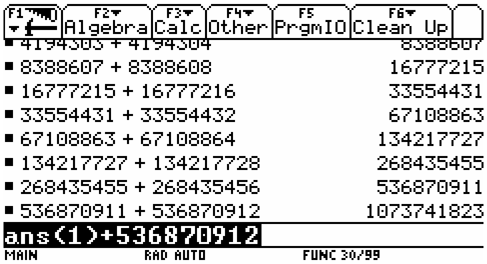


FIGURE 5

Thus after seven days, 127 pennies or $1.27 was earned. In FIGURE 2, the students observed that the entry 1023 represented the sum of the initial ten terms of the series in the sense that $2^0+2^1+2^2+2^3+2^4+2^5+2^6+2^7+2^8+2^9=2^{10}-1=1,024-1=1,023$ pennies. So far, the parents appear to be well in control of the finances. If we now consider the next ten terms of the series, we obtain the following sum: $2^{10}+2^{11}+2^{12}+2^{13}+2^{14}+2^{15}+2^{16}+2^{17}+2^{18}+2^{19}=1,047,552.$ We are asserting that during days 11-20 of the month, the parent's layout was 1,047,552 pennies or $1,0475.52! Thus one has a cumulative total of 1,048,575 pennies or $10,485.75. Finally adding the totals for days 21-30 is asserting the following computation: $2^{20}+2^{21}+2^{22}+2^{23}+2^{24}+2^{25}+2^{26}+2^{27}+2^{28}+2^{29}=1,072,693,248.$ One is conveying the sum of 1,072,693,248 pennies or $10,726,932.48 is the allotment for the last ten days. Adding the three totals, we obtain $1,023+1,047,552+1,072,693,248=1,073,741,823$ pennies or $10,737,418.23! No sweepstakes for this youngster! FIGURES 6-7 display the relevant calculations:

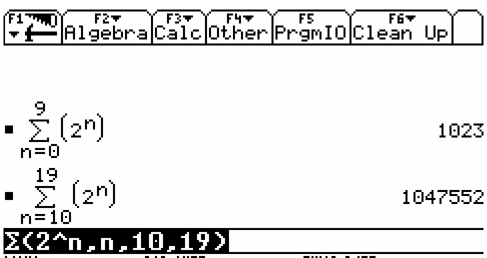


FIGURE 6

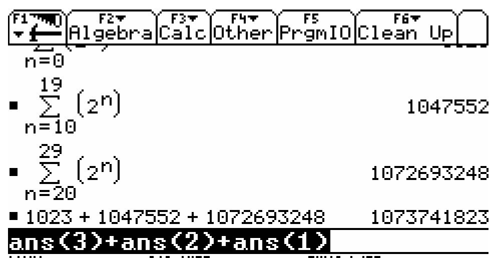


FIGURE 7

This problem is revealing for the following reason: No matter how large a family budget, if enough time elapses, the sum eventually will exceed the budget! To cite an example, if the family allocated a $1,000 budget for this project, then after seventeen days, this budget would have been exceeded; for 131,071 pennies or $1,310.71 has been given to the child! One thus is introduced to the notion that some geometric series do not possess a sum and the idea of divergent series is considered.

Activity 2: The Problem of Tartaglia: In 1556, the mathematician Tartaglia asserted that the sequence $1+2+4, 1+2+4+8, 1+2+4+8+16, \ldots$ was alternately prime and composite. If one proceeds far enough into the sequence in which we double the last term in the preceding sum to obtain the next term, observe the following in FIGURES 8-10:

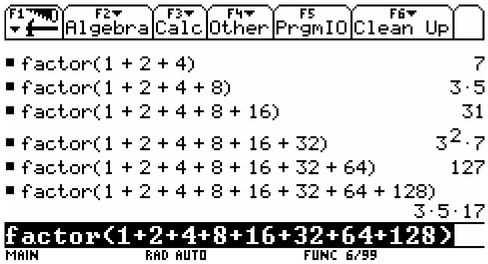


FIGURE 8

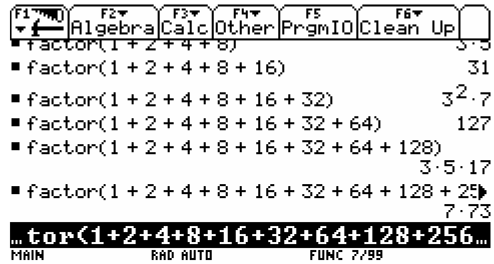


FIGURE 9

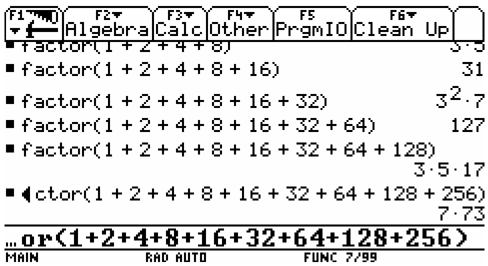


FIGURE 10

We observe that two consecutive composite integers are obtained; namely 255 (divisible by 5) and 511 (divisible by 7). Of course, in 1556, the technology was far less developed so that 511 would have been considered fairly large! If one extended the activity, a legitimate inquiry might entail when the next prime entry would occur. Continuing the process, the reader would discover that the next six prime outputs occur when one considers the sums

$$\sum_{n=0}^{12} 2^n = 2^{13} - 1 = 8{,}191$$

$$\sum_{n=0}^{16} 2^n = 2^{17} - 1 = 131{,}071$$

$$\sum_{n=0}^{18} 2^n = 2^{19} - 1 = 524{,}287$$

$$\sum_{n=0}^{30} 2^n = 2^{31} - 1 = 2{,}147{,}483{,}647$$

$$\sum_{n=0}^{60} 2^n = 2^{61} - 1 = 2{,}305{,}843{,}009{,}213{,}693{,}951$$

$$\sum_{n=0}^{88} 2^n = 2^{89} - 1 = 61{,}897{,}001{,}964{,}269{,}013{,}744{,}562{,}111$$

These can be verified with the aid of a graphing calculator equipped with a CAS (Computer Algebra System) such as the VOYAGE 200. See FIGURES 11-13 below:

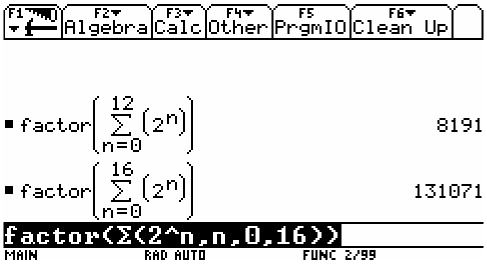


FIGURE 11

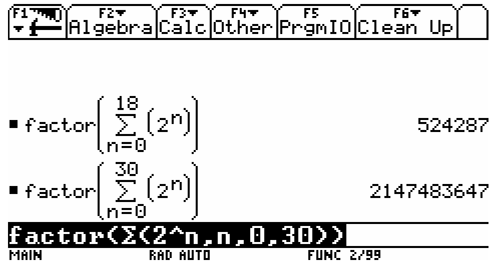


FIGURE 12

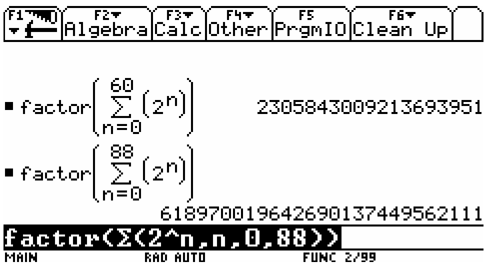


FIGURE 13

Martin Mersenne, the amateur mathematician in the seventeenth century considered positive primes p such that $2^p - 1$ is likewise prime. Then $2^p - 1$ is called a ***Mersenne Prime.*** Mersenne primes are important; for they lead to even perfect numbers. A ***perfect number*** is one that is equal to the sum of its aliquot (proper) divisors. For example, 6 and 28 are perfect numbers; for $6 + 1 + 2 + 3 \textit{ and } 28 = 1 + 2 + 4 + 7 + 14.$ Euclid proved that if $P = \left(2^{p-1}\right) \cdot \left(2^p - 1\right)$ where both $p \,\&\, 2^p - 1$ are primes, then P is an even perfect number. The great Swiss mathematician Leonhard Euler (1707-1783) proved the more difficult converse. The initial two even perfect numbers 6 and 28 correspond to the primes $p = 2 \,\&\, p = 3$ respectively. Currently forty-four even perfect numbers are known.

Activity 3: The Rows of Pascal's Triangle of the Form $2^n - 1$:

Recall the first fifteen rows of Pascal's Triangle where the apex of the triangle is considered Row 0. The rows of the triangle are as follows:

1

1 1

1 2 1

1 3 3 1

1 4 6 4 1

1 5 10 10 5 1

1 6 15 20 15 6 1

1 7 21 35 35 21 7 1

1 8 28 56 70 56 28 8 1

1 9 36 84 126 126 84 36 9 1

1 10 45 120 210 252 210 120 45 10 1

1 11 55 165 330 462 462 330 165 55 11 1

1 12 66 220 495 792 924 792 495 220 66 12 1

1 13 78 286 715 1287 1716 1716 1287 715 286 78 13 1

1 14 91 364 1001 2002 3003 3432 3003 2002 1001 364 91 14 1

1 15 105 455 1365 3003 5005 6435 6435 5005 3003 1365 455 105 15 1

Observe that the entries in the 1st, 3rd, 7th, and 15th rows of the triangle have only odd entries. Note that each of these integer rows is of the form $2^n - 1$. One additionally observes that each of the entries in the triangle is obtained from the entries in the previous row in a special manner. For example, the third entry in the 7th row of the triangle, 21, is obtained by adding the second and third entries in the 6th row; namely 6 and 15. Finally, each row in the triangle from left to right furnishes one with the number of subsets of a set having 0, 1, 2, 3,…, n elements where n is the number of elements in the given row. Hence in Row 5 of the triangle, the binomial coefficients 1, 5, 10, 10, 5, 1 denote the number of 0, 1, 2, 3, 4, and 5 element subsets possessed by a set having five elements. In particular, one has 10 three-element subsets. Observe that the number of proper subsets of a set having n elements is precisely $2^n - 1$. A ***proper subset*** is any subset apart from the full set.

Activity 4: The Base Two (Binary) Representation of $\sum_{k=0}^{n} 2^k = 2^n - 1$:

Consider each of the following computations:

$2^0 + 2^1 = 1 + 2 = 3 = 11_2$

$2^0 + 2^1 + 2^2 = 1 + 2 + 4 = 7 = 111_2$

$2^0 + 2^1 + 2^2 + 2^3 = 1 + 2 + 4 + 8 = 15 = 1111_2$

$- -$

In general, in base two, $2^n - 1 = (1111...)_2$ where we have n ones in the binary representation.

Conclusion

In this set of activities, participants were engaged in discovery and to their delight witnessed that a single formula can be applicable in various settings throughout the curriculum. This type of engagement is essential to learning new mathematics and is in keeping with being an active rather than passive learner and comports with the NCTM Curriculum Standards. The reader is invited to discover additional activities attributed to this formula. Numerous ideas and excursions are available on The World Wide Web to aid in discovery-based learning. Two particularly impressive Web Sites are MathWorld and The On-Line Encyclopedia of Integer Sequences referenced below.

References

Neil J.A. Sloane (2007) *The On-Line Encyclopedia of Integer Sequences*, Murray Hill, NJ: AT&T Research

Eric W. Weisstein (2007) *MathWorld-A Wolfram Resource*, Champaign, IL: Wolfram Research, Inc.

495's Interesting Property

Robert Mitchell,

Professor of Mathematics (retired),

Rowan University

Introduction

Teachers looking for surprising number properties to show their students will enjoy this number trick. It involves 495, and it doesn't matter what number you choose, as long as it is a three digit number. For those looking for a link to the NJCCCS in Mathematics, this addresses Numerical Operations (4.1 - B), and Patterns (4.3 – A).

Procedure

Select any three digit positive integer where not all the digits are the same. Arrange the three digits so that the number is as large as possible. Arrange the three digits so that the number is as small as possible. Subtract the smaller number from the larger one and keep repeating the procedure with the difference. Eventually the difference will be 495.

Examples

Starting with 898, the first difference is 988-889=099 and the second difference is 990-099=891. The third difference is 981-189=792 and so on. The sequence of differences is{099,891,792,693,594,495,495,495,…..}. Starting with 871, the sequence of differences is {693,594,495,495,….}. Starting with 383, the sequence of differences is {495,495,495,…….}. The problem can be solved by running a computer program that exhaust all the possibilities, but the purpose of this paper is to present a proof that is computer free.

The General Case

Note that each difference in the examples contains the number 9. Let's look at the general case. Let ABC be a three digit positive integer where not all the digits are the same. If A is the largest integer and C is the smallest integer, then ABC-CBA=X9Y, where X=A-1-C and Y=10+C-A. Since A is greater than C, then X is a non-negative single digit, Y is a positive single digit and 9 is the largest digit in each difference.

9BC-CB9=P9Q where P=8-C and Q=1+C. If C is the minimum digit in a difference, then the minimum digit in the next difference is min(8-C,1+C). If C=0 then min(8,1)=1 and if C=1 then min(7,2)=2. If C=2 then min(6,3)=3 and if C=3 then min(5,4)=4. If C=4 then min(4,5)=4; therefore, if the minimum digit in the first difference is 0, then the sequence of minimum digits in the differences is {0,1,2,3,4,4,4,4.….} as in the first example. If the minimum digit in the first difference is 3, then the sequence of minimum digits in the differences is {3,4,4,4.…} as in the second example and if the minimum digit in the first difference is 4 then the sequence of minimum digits in the differences is {4,4,4,4,4.…..} as in the third example. For the other six cases the sequences are {1,2,3,4,4,4,….}

{2,3,4,4,4,4,….} {5,3,4,4,4,4,……..} {6,2,3,4,4,4,….} {7,1,2,3,4,4,….} and {8,0,1,2,3,4,4,4,…}

In every case the maximum digit is 9 and the minimum digit is eventually 4.

9B4-4B9=495 and 954-459=495. Each term in the sequence of differences is 495 from then on.

Conclusion

For readers interested in extending this to other bases, the following can be proven by proofs similar to the above:

In base 6 the number is 253 and in base 12 the number is 5E6.

In base 2N the number is XYN where X=N-1 and Y=2N-1

In base 5 the sequence will eventually oscillate between 143 and 242

In base 7 the sequence will eventually oscillate between 264 and 363.

In base 2N+1 the sequence will eventually oscillate between XYZ and NYN

where X=N-1, Y=2N and Z=N+1.

For four digit numbers in base 10, the number is 6174.

Robert Mitchell earned his degrees at the University of Texas and taught at Glassboro State College/Rowan University for 32 years retiring in 1997. He taught courses ranging from Basic Skills to graduate courses and enjoyed the variety. One of his avocations is Lewis Carroll and he has been guest lecturer on Carroll and his contribution to mathematical logic at numerous high schools and colleges over the years as well as meetings of the British Society of the History of Mathematics in Lancaster and Winchester. You can reach him at: 107 York Avenue, West Cape May, NJ 08204-1259

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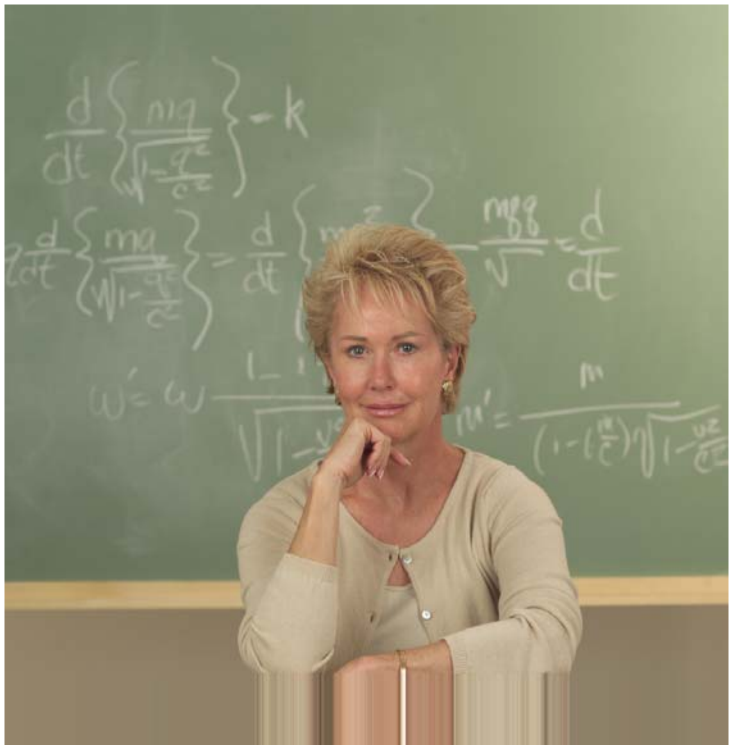

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Registration for both days (Thursday and Friday)	___$150	___$180
Members –ONLY special #1 includes Wednesday Evening Dinner, Wednesday night Hotel room for the Marriott, Registration for Thursday Conference (Thursday night reception free)	___$225	
Members –ONLY special #2 includes Wednesday Evening Dinner, Wednesday and Thursday night Hotel room for the Marriott, Registration for both days of the conference (Thursday night reception free)	___$325	

(2) Select MEAL SESSIONS YOU WISH TO PURCHASE

THURSDAY BREAKFAST -TBA	***___$15***
THURSDAY LUNCH -Sponsored by EAI	***___$30***
FRIDAY BREAKFAST WILL BE A BENEFIT All proceeds will benefit the AMTNJ scholarship fund Minimum donation of $20 per ticket. Please note donation amount – tax deductible receipt will be sent	***_____$***
FRIDAY LUNCH Sponsored by Texas Instruments	***___$30***

Note changes to the 2007 year conference include:
NO tickets required for any sessions other than the meal sessions.
All sessions are first come first serve- no exceptions no reservations

Total AMOUNT DUE AMTNJ
$____________

Payment Method
Purchase Order #_______ (can be marked to follow)
Personal Check ____________

Please note: if payment option is NOT checked- your school will be invoiced for a PO – but the responsibility lies with the participant

III. RETURN to:

AMTNJ PO Box 7 Glassboro, NJ 08028 Or Fax to: 856-358-4374
or ...Email them as attachments with a receipt requested: or for more information email: amtnj@juno.com)

PLEASE NOTE: AMTNJ POLICY DOES NOT ACCEPT CANCELLATIONS AFTER ONE WEEK PRIOR TO ANY CONFERENCE