AMTNJ
37th Annual Contest
Solutions Key

1) $\frac{100(x-yz)}{x}$ or any equivalent form

2) 11 The triangles have sides of: (6,6,6) (6,6,5) (6,6,4) (6,6,3) (6,6,2) (6,6,1) (6,5,5) (6,5,4) (6,5,3) (6,5,2) and (6,4,3)

3) 3 $13^1=13,\ 13^2=169,\ 13^3=2197,\ 13^4=28561$, The units digits are a cyclic group of 4. Therefore the units digit of 13^{13} is 3.

4) -2012 If 0 is the middle digit then there are 1006 on either side, so the smallest even number is -2012.

5) $\binom{9}{4}=126$ Any combination of the nine different digits, excepting zero can be arranged in rising order.

6) 3

$$\log_7(x+4)+\log_7(x-2)=1$$
$$(x+4)(x-2)=7$$
$$\therefore x=3$$

7) 108 The median divides the triangle into two triangles with equal area. Therefore the area of each "half" is 54.

8) f(4) = 21

If x=4, 2f(4)-f(-3)=14
if x=-3, 2f(-3)-f(4)=35
solving the system for f(4) yields f(4)=21.

9) (5, 15) and (9, 13)

The product of all the roots is 62500.

(x+yi)(x-yi)(y+xi)(y-xi)=$(x^2+y^2)^2$

and $(x^2+y^2)^2 = 62500$ yields

(5, 15) and (9,13) if x<y.

10) 15 Drawing the picture shows that $\triangle AYZ$ is 1/6 of 1/2 the parallelogram, which is 1/6 of 90 or 15.

11) 1006 or 1008

$$\frac{2013!}{1006!1007!}+\frac{2013!}{1005!1008!}=\binom{2014}{x}$$

$$2013!(\frac{2014}{1006!1008!})=\frac{2014!}{1006!1008!}$$

therefore x= 1006 or 1008

12) D = 7

SEND =	9567
+MORE	+1085
MONEY	10652

Therefore D=7

13) $\frac{81}{3+2\sqrt{2}}=81(3-2\sqrt{2})$

The sides of the triangle are x, x, and 18-2x. Since $18-2x=x\sqrt{2}$, then $x=\frac{18}{2+\sqrt{2}}$

14) $2-\sqrt{3}$

$$\tan(15°)=\tan(\frac{30}{2})=\sqrt{\frac{1-\cos(30)}{1+\cos(30)}}=\sqrt{\frac{1-\frac{\sqrt{3}}{2}}{1+\frac{\sqrt{3}}{2}}}=$$

$$\sqrt{\frac{2-\sqrt{3}}{2+\sqrt{3}}}=\frac{1}{2+\sqrt{3}}=2-\sqrt{3}$$

15) $\frac{1}{8}$

$$A_f=A_0(\frac{1}{2})^{kt}\rightarrow\frac{1}{4}A_0=A_0(\frac{1}{2})^{8k}\rightarrow k=\frac{1}{4}$$

$$\therefore A_f=A_0(\frac{1}{2})^{.25(12)}=\frac{1}{8}A_0$$