

**Directions:**

- Your answers should be in the form specified in the problem. Approximate answers must be at least three decimal places rounded or truncated (ex:  $\frac{2}{3} \approx 0.666$  or  $0.667$ ), and exact answers must be in simplest form (ex:  $\frac{5}{15}$  will not be accepted for  $\frac{1}{3}$ , and  $\sqrt[3]{48}$  will not be accepted for  $2\sqrt[3]{6}$ ). When the desired form is specified in a problem, any other form of the answer will not receive credit.
  - You may only use calculators that are permitted on the SAT I.
  - You may write on this contest and use additional paper you receive from your teacher, but you should write your answers on the **Individual Student Cover Page** to be official and receive credit.
  - You will have exactly 45 minutes to complete the problems in this contest. Work quickly and with accuracy.
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**Problems:**

1. Find the area of the triangle that has vertices  $(-1, 3)$ ,  $(1, 1)$  and  $(3, 5)$ .
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2. Given three consecutive odd integers  $a$ ,  $b$  and  $c$  such that  $0 < a < b < c$ . What is the value of  $a^2 - 2b^2 + c^2$ ?
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3. The measure of angle  $A$  in a scalene triangle  $AMT$  is  $42^\circ$ . The bisectors of angles  $M$  and  $T$  intersect in point  $N$  inside the triangle. Find the measure of angle  $MNT$ .
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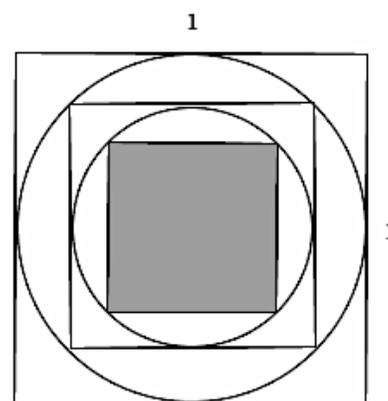
4. What is the exact value of  $\frac{3^{2017} - 3^{2014}}{3^{2017} + 3^{2014}}$  in its simplest form?
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5. Find the exact value of  $k$  if  $\frac{2x^4 - (kx)^3 + x^2 - 4}{x+1} = p(x) - \frac{7}{x+1}$  for some polynomial  $p(x)$ .
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6. Given a right triangle with legs that have lengths  $m$  and  $n$ , and a hypotenuse of length  $n + 1$ . If  $m$  and  $n$  are integers and  $n \leq 60$ , how many such triangles exist?
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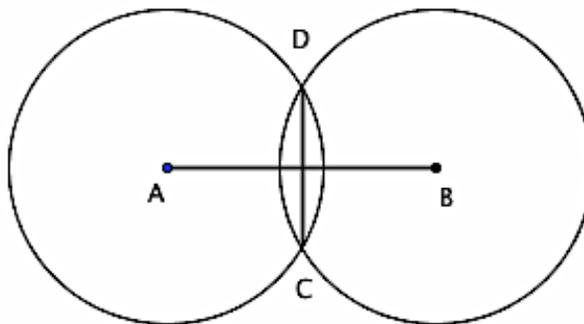
7. Find the exact value of the infinite continued fraction  $\frac{2}{2 + \frac{2}{2 + \frac{2}{2 + \frac{2}{2 + \dots}}}}$ .
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8. Find the area of the shaded inner square in the diagram shown, where the outer square has side 1, and the two circles are tangent to the sides of the squares.



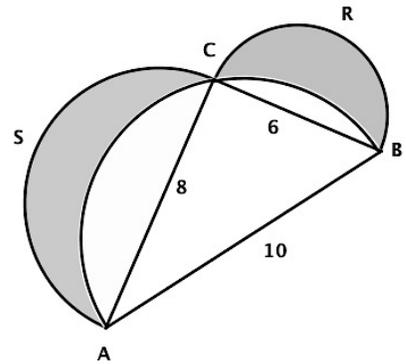
9. An *arithmetic sequence* is a sequence of numbers that differ by a constant, called the *common difference*. Given an arithmetic sequence in which the sum of the first  $n$  terms is 216. If the first term in this sequence is  $n$  and the  $n^{\text{th}}$  term is  $2n$ , find the common difference of the sequence.
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10. Two congruent circles with centers  $A$  and  $B$  have a radius of length 6 units. The circles intersect at points  $C$  and  $D$ , and chord  $\overline{CD}$  also has a length of 6 units. If the area of the region enclosed by the two circles is written as  $a\pi + \sqrt{b}$ , what is the value of  $a + b$ ?



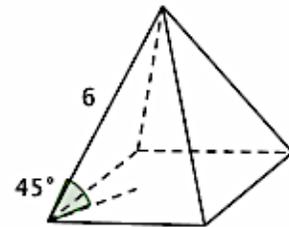
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11. A *partition* of a positive integer  $n$  is a way of writing  $n$  as the sum of positive integers. When counting partitions, rearrangements of integers in a sum are counted once. For example, 4 has 5 partitions: 4, 3+1, 2+2, 2+1+1, and 1+1+1+1. Note that 2+1+1, 1+2+1 and 1+1+2 are counted as one partition. How many partitions does 7 have?
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12. Triangle  $ABC$  is inscribed in a semicircle with diameter  $\overline{AB}$ , with  $AB = 10$ ,  $AC = 8$  and  $CB = 6$ . If semicircles  $ASC$  and  $CRB$  have diameters  $\overline{AC}$  and  $\overline{CB}$  respectively, what is the area of the shaded region in the figure on the right?



13. How many real solutions does the equation  $e^{0.12x} - x^{12} = 0$  have?
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14. In a regular square pyramid, a lateral edge is 6 cm long. Find the exact volume of the pyramid if each lateral edge makes a  $45^\circ$  angle with its projection in the base.



15. Sections of Portland, Oregon are laid out in rectangular blocks. One such section consists of eleven east-west streets intersecting with seven north-south streets as shown. Find the number of different paths, each 16 blocks long, in going from the south-west corner  $A$ , to the north-east corner  $B$ .

