THE 2017 AMTNJ 27TH ANNUAL CONFERENCE

GROWTH MINDSTS IN MATHEMATICS AND IMPLEMENTING THE NEW

NEW JERSEY MATHEMATICS STANDARDS

OCTOBER 26-27, 2017

THE NATIONAL CONFERENCE CENTER AND THE HOLIDAY INN HOTEL

EAST WINDSOR, NJ

DATE AND TIME OF THE PRESENTATION: FRIDAY, OCTOBER 27, 2017

11:30 A.M. – 1:00 P.M.

LOCATION: CONFERENCE ROOM D

SESSION NUMBER: 69

TITLE OF PRESENTATION: USING MATHEMATICAL RECREATIONS TO BALANCE PROCEDURAL FLUENCY AND CONCEPTUAL UNDERSTANDING

PRESENTER: JAY L. SCHIFFMAN

ROWAN UNIVERSITY

<u>USING MATHEMATICAL RECREATIONS TO BALANCE PROCEDURAL FLUENCY AND CONCEPTUAL UNDERSTANDING</u>

JAY L. SCHIFFMAN

ROWAN UNIVERSITY

<u>ABSTACT:</u> Accomplishing both procedural fluency and conceptual understanding is not a bogus dichotomy. In this hands-on workshop, participants will be engaged in puzzles and other mathematical recreations to balance skills and concepts. The mathematical recreations will be selected from the fields of number and operations, discrete mathematics, algebra and geometry.

SOME PROBLEMS AND DISCUSSION ACTIVITIES:

I. Use both inductive reasoning (five cases) and then deductive reasoning to solve the following number puzzle employing the given directives:
a. Pick any number.
b. Add 221 to the given selected number.
c. Multiply the sum by 2652.
d. Subtract 1326 from your product.
d. Divide your difference by 663.
e. Subtract 870 from your quotient.
f. Divide your difference by 4.
g. Subtract the original number from your quotient.

II. (a). Determine all the primes in the following sequence:

$$\left\{\begin{array}{l} 1;\ 31;\ 331;\ 3331;\ 33331;\ 333331;\ 3333331; 33333331; \\ 333333331; 3333333331;\ 33333333331;\ 333333333331; \\ 3333333333331;\ 33333333333331;\ 333333333333331; \\ 3333333333333331;\ 33333333333333331;\ 333333333333333331 \end{array}\right\}$$

The general formula for this sequence is $p(n)=\dfrac{10^n-7}{3}$.

(b). Determine all the primes in the *repunit* sequence:

$$\left\{\begin{array}{l} 11;111;1111;11111;111111;1111111;11111111; \\ 111111111;1111111111;11111111111;111111111111; \\ 1111111111111;11111111111111;111111111111111; \\ 1111111111111111;11111111111111111;111111111111111111; \\ 1111111111111111111 \end{array}\right\}$$

The general formula for this sequence is $p\left(n\right)=\dfrac{10^n-1}{9}$.

III. Show that one has a magic square of order 3 whose entries are consecutive primes. The magic square is shown below:

1480028159	**1480028153**	**1480028201**
1480028213	**1480028171**	**1480028129**
1480028141	**1480028189**	**1480028183**

IV. In 1975, the largest pair of known twin primes was 140737488353699 and 140737488353701. Using the TI and MATHEMATICA technologies, do each of the following:
(a). Prove that the above integers are indeed prime in several different ways via our technologies.

(b). By employing the Next Prime option, find a larger pair of twin primes disproving that the above two primes do not form the largest twin prime pair.

V. <u>A Fun Activity with the Fibonacci sequence.</u>

Consider the sum of any six consecutive terms in the Fibonacci sequence. Form the sum and divide by four. Try this with three different numerical data sets. Form a conjecture. Can you prove your conjecture? Repeat this problem for the sum of ten consecutive terms in the Fibonacci sequence. Form the sum and divide by eleven. Next consider the sum of any fourteen consecutive terms in the Fibonacci sequence. Form the sum and divide by twenty-nine.

VI. <u>Geometry and the Fibonacci sequence.</u>

Consider any four consecutive terms in the Fibonacci sequence. First form the product of the first and fourth terms. Take twice the product of the second and third terms. Finally take the sum of the squares of the second and third terms in your sequence. Try to relate this to a theorem in plane geometry, conjecture based on several examples, and try to substantiate your conjecture.

VII. Consider the sum of two unit fractions with consecutive even denominators such as $\frac{1}{2}+\frac{1}{4}$**. Do this for the first fifteen iterations. (i.e. Consider** $\frac{1}{4}+\frac{1}{6}, \frac{1}{6}+\frac{1}{8},$ **etc.) What do you notice when considering the numerators and denominators in each of these sums? Repeat this with the sum of two unit fractions with consecutive odd denominators such as** $\frac{1}{3}+\frac{1}{5}$**. Repeat for fifteen iterations. What do you notice about the numerators and denominators in the sums? Do you see any connections to geometry?**

VIII. More computer time has been devoted to The Collatz Problem (also known as the $3X+1$ **problem) than any other in number theoretic lore. The rules are very simple. If an integer is even, divide by two. If an integer is odd, triple and add one. Repeat this process on each new integer obtained. After finitely many steps, the sequence will converge to one. This conjecture has been open since 1937 and verified for all integers** $\leq 5\cdot 10^{18}$**. This easily posed problem that has fascinated yet eluded and frustrated mathematicians for more than seventy-five years.**
Determine the number of iterations needed to reach 1 for each of the following positive integers applying The Collatz Sequence rules: (a). 18 (b). 128, (c). 25

SOLUTIONS TO PROBLEMS AND ACTIVITIES:

I. In inductive reasoning, we reason to a general conclusion via the observations of specific cases. The conclusions obtained via inductive reasoning are only probable but not absolutely certain. In contrast, deductive reasoning is the method of reasoning to a specific conclusion through the use of general observations. The conclusions obtained through the use of deductive reasoning are certain. In the following number puzzle, we employ the five specific numbers 5, 23, 12, 10, and 85 to illustrate inductive reasoning and then employ algebra to furnish a deductive proof. The puzzle and the solutions are provided below:

Pick any Number.	5	23	12	10	85	n
Add 221 to the given selected number.	226	244	233	231	306	$n+221$
Multiply the sum by 2652.	599352	647088	617916	612612	811512	$2652n+586092$
Subtract 1326 from your product.	598026	645762	616590	611286	810186	$2652n+584776$
Divide your difference by 663.	902	974	930	922	1222	$4n+882$
Subtract 870 from your quotient.	32	104	60	52	352	$4n+12$
Divide your difference by 4.	8	26	15	13	88	$n+3$
Subtract the original number from your quotient.	3	3	3	3	3	3

The answer we obtain is always 3. We next deploy the calculator to show the inductive cases in **FIGURES 1-10** and the deductive case in **FIGURES 11-12:**

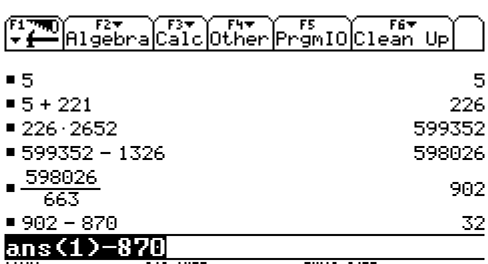


FIGURE 1

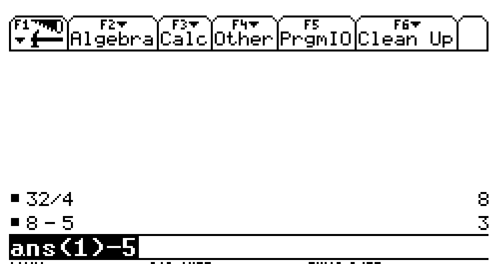


FIGURE 2

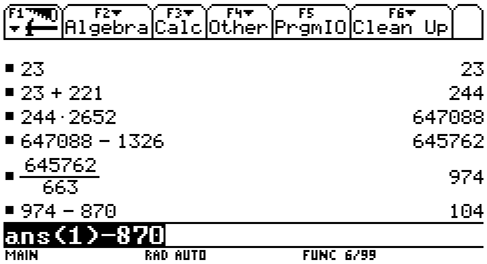


FIGURE 3

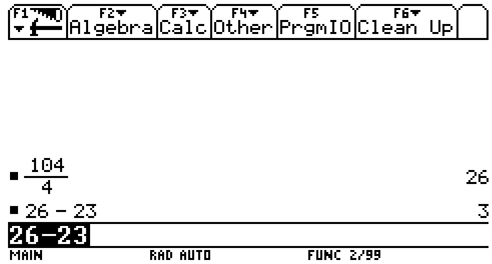


FIGURE 4

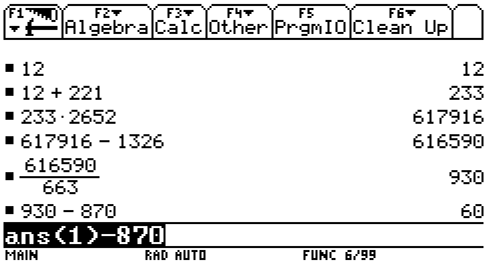


FIGURE 5

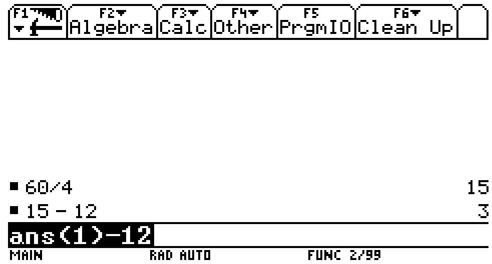


FIGURE 6

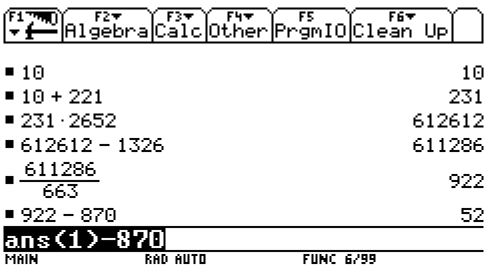


FIGURE 7

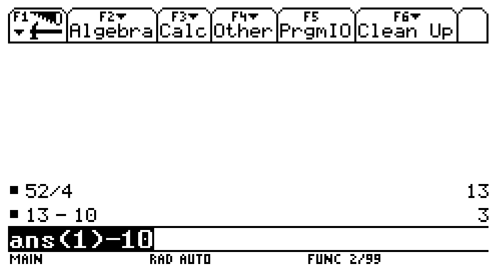


FIGURE 8

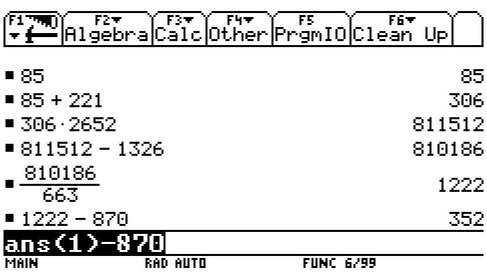


FIGURE 9

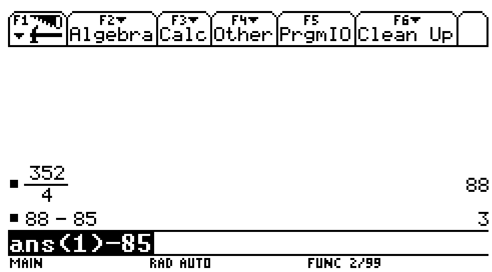


FIGURE 10

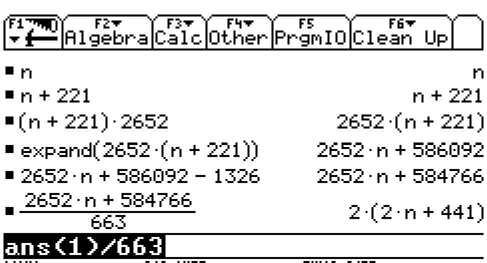


FIGURE 11

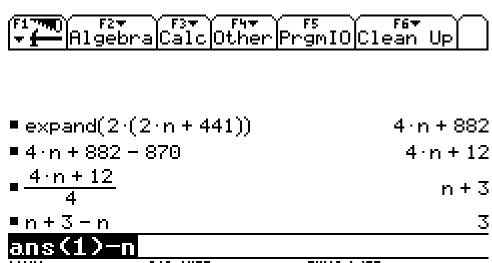


FIGURE 12

II. (a). We determine the primes in the sequence that consists of $n = 0, 1, 2, 3, ..., 18$ 3's followed by a 1 with the general $n-th$ term given by $a_n = \dfrac{10^n - 7}{3}$. We appeal to **FIGURES 13-21:**

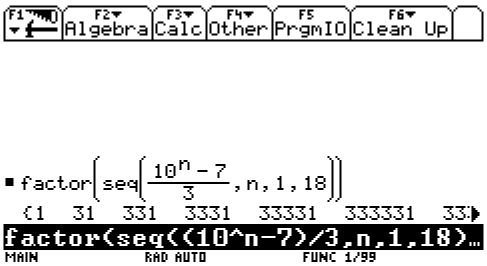


FIGURE 13

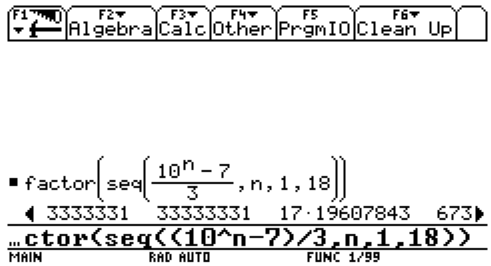


FIGURE 14

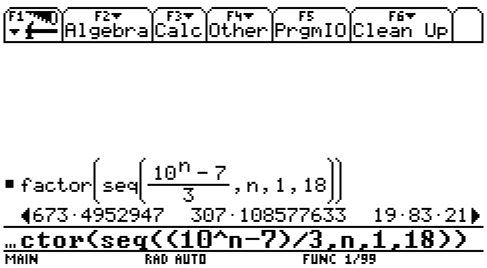


FIGURE 15

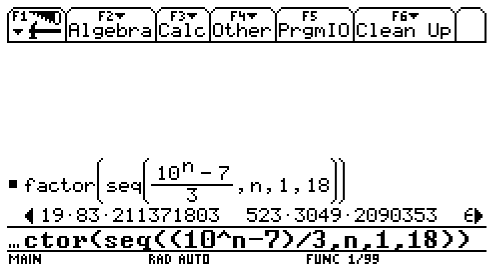


FIGURE 16

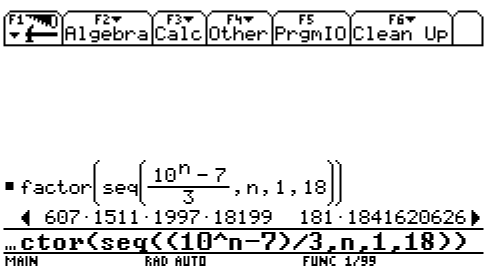


FIGURE 17

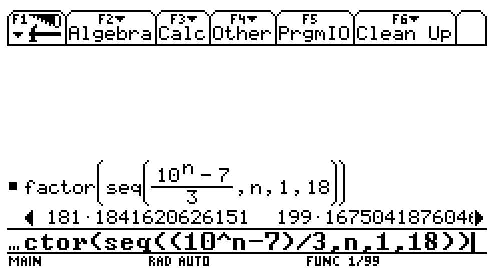


FIGURE 18

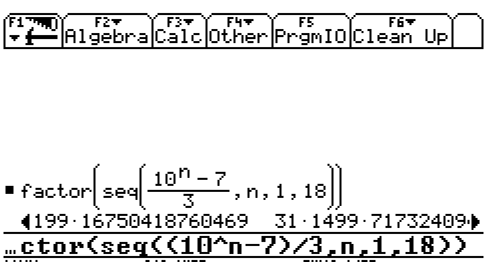


FIGURE 19

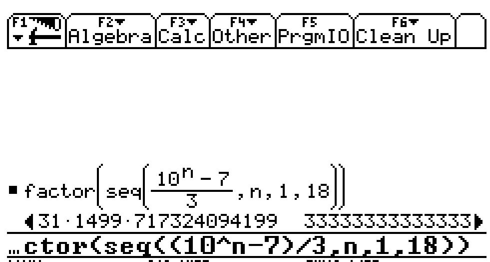


FIGURE 20

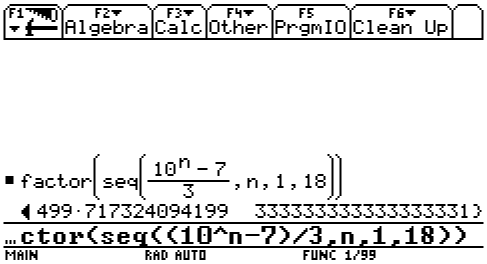


FIGURE 21

Without resorting to the general $n-th$ term of the sequence, we factor each of the integers separately in **FIGURES 22-25:**

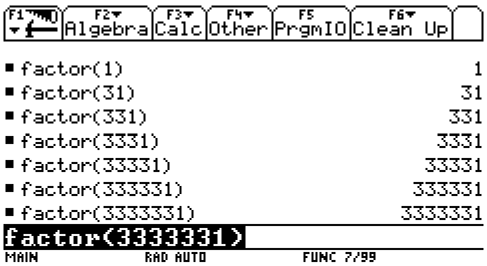


FIGURE 22

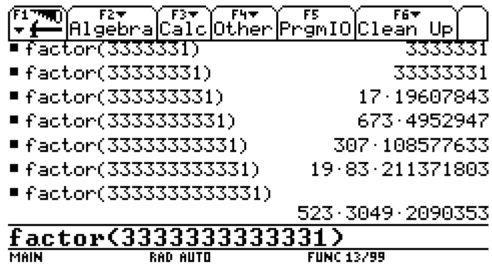


FIGURE 23

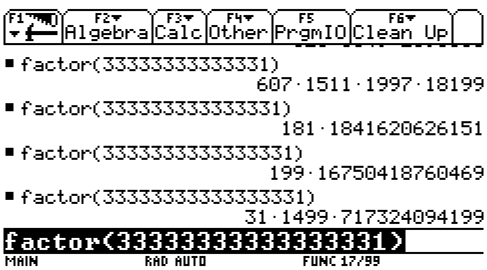


FIGURE 24

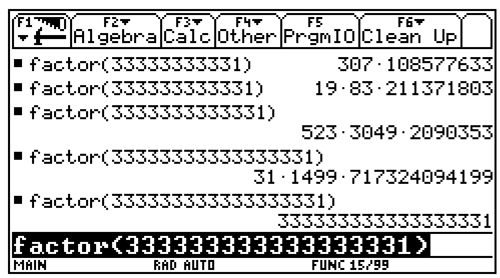


FIGURE 25

Clearly it is easier to display the outputs without resorting to a formula. It turns out the above sequence is prime for $n=2,3,4,5,6,...,18,$ *and* 40 among the small values.

(b). We determine the primes in the repunit (repeated digit) sequence consisting of $2,3,4,...,19$ 1's. Our formula for the general $n-th$ term can be given by $a_n = \dfrac{10^n - 1}{9}$.

We display in **FIGURES 26-36** the prime outputs:

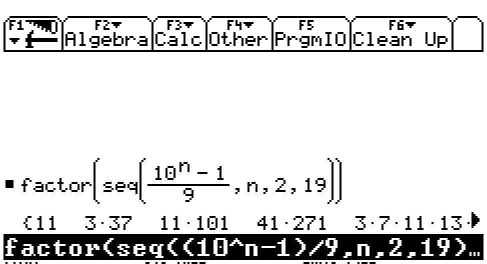


FIGURE 26

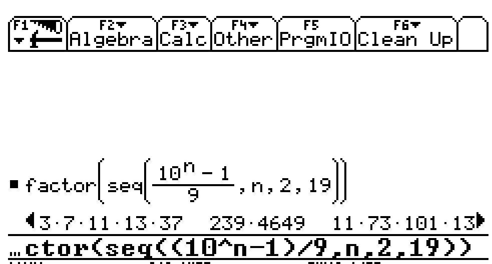


FIGURE 27

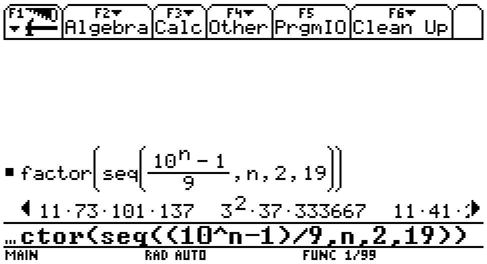


FIGURE 28

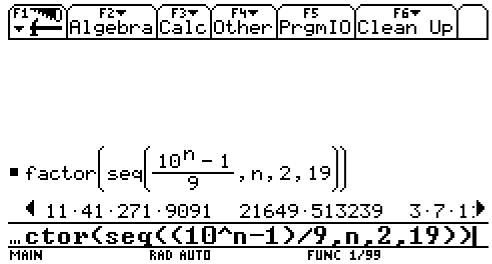


FIGURE 29

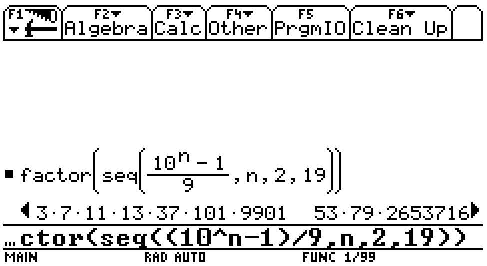


FIGURE 30

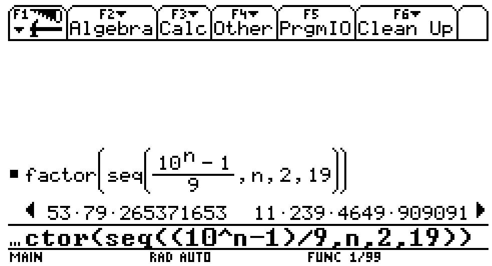


FIGURE 31

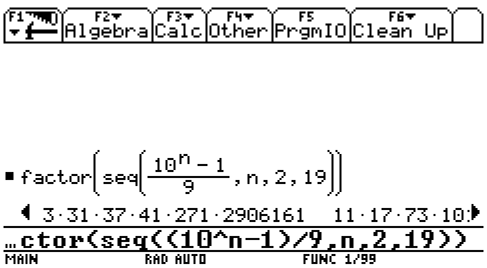


FIGURE 32

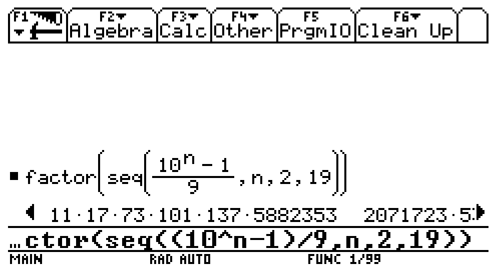


FIGURE 33

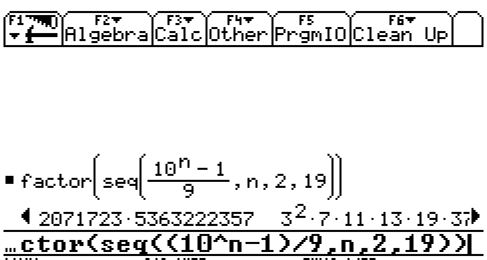


FIGURE 34

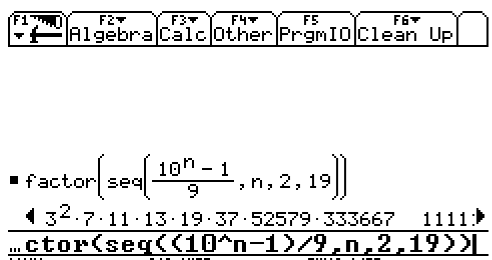


FIGURE 35

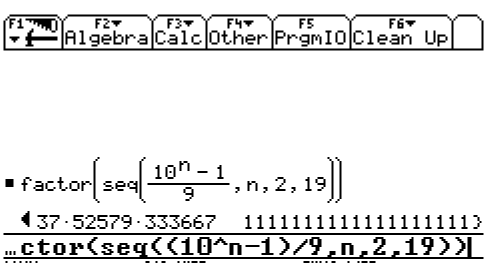


FIGURE 36

It is easier to display the outputs individually as in **FIGURES 37-40:**

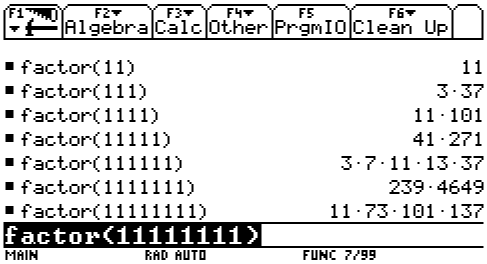


FIGURE 37

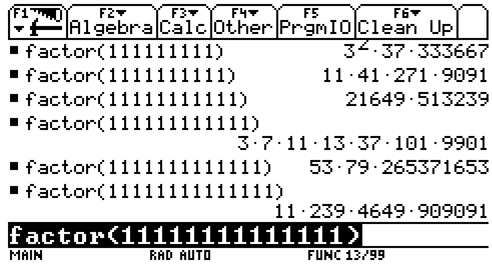


FIGURE 38

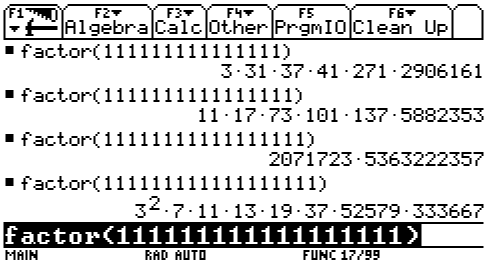


FIGURE 39

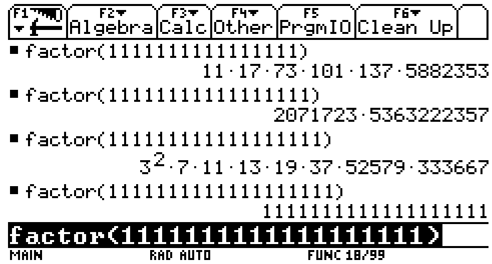


FIGURE 40

It turns out that the result above is prime for $n = 2.$ If $n = 19, 23, 317, 1031, 49081,$ *and* 86453, then repunit sequences consisting of those number of 1’s are likewise prime.

III. A **magic square is** a square such that the sum of the entries in every row, every column, and along both diagonals is always identical. This common sum is called the **magic constant.** Magic squares of size $n \times n$always exist for $n \geq 3.$ There is no 2×2 square. The magic constant (also called the **magic sum** for an $n \times n$ magic square) is given by the formula $\frac{n \cdot \left(n^2 + 1\right)}{2}$.

The following magic square of order 3 has nine entries each of which is a prime: (In addition, these primes are consecutive!)

1480028159	1480028153	1480028201
1480028213	1480028171	1480028129
1480028141	1480028189	1480028183

We first demonstrate that each of these integers is indeed prime in **FIGURES 41-42:**

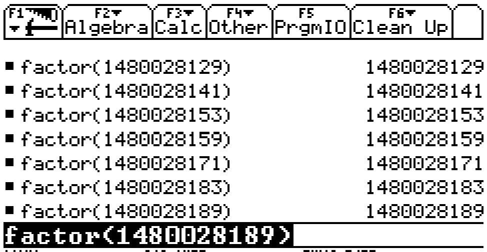


FIGURE 41

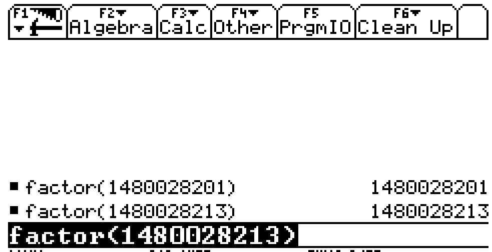


FIGURE 42

On Page 435 of the TI-89 manual, a program for the Next Prime is given. In **FIGURES 43-44,** we view the Program Ed (Program Editor) from the APPS MENU and **in FIGURE 45,** we view the program after pressing ENTER in **FIGURE 44:**

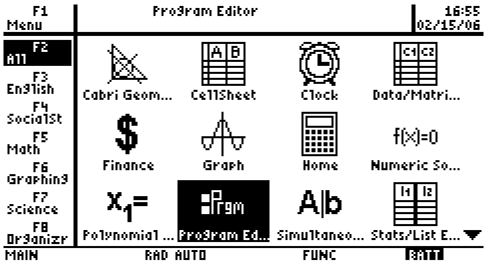


FIGURE 43

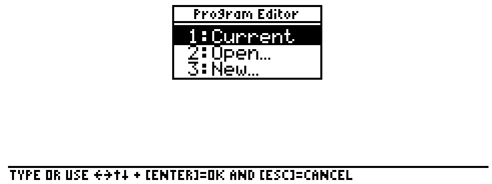


FIGURE 44

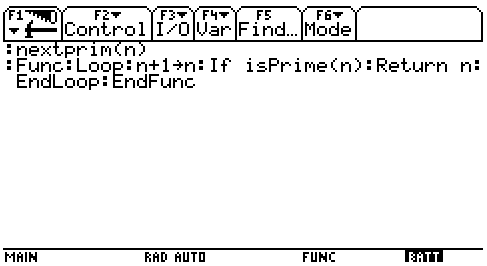


FIGURE 45

In **FIGURE 46,** we view the program in the Variables Link folder which is 2nd (-) (VAR LINK):

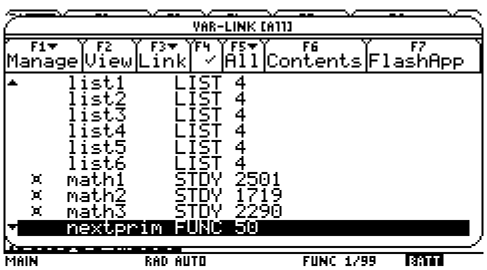


FIGURE 46

To cite a simple example, 11 is the prime successor to the prime 7 as we view in **FIGURE 47:**

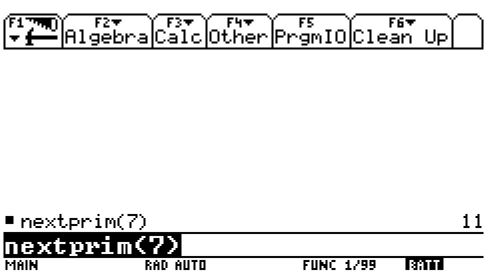


FIGURE 47

We next employ the Next Prime program to show that these nine primes are consecutive in **FIGURES 48-49:**

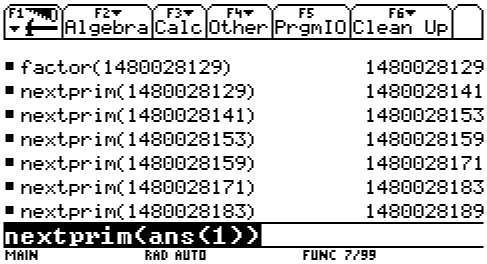


FIGURE 48

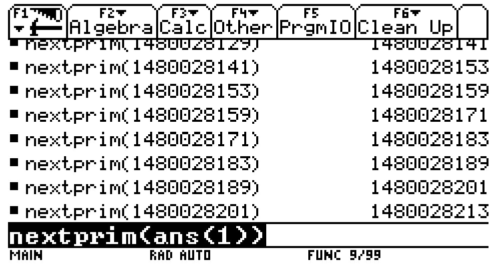


FIGURE 49

We next show that the configuration is indeed a magic square with the row sums in **FIGURES 50-51,** the column sums in **FIGURES 52-53** and the diagonal sums in **FIGURES 54-55** respectively:

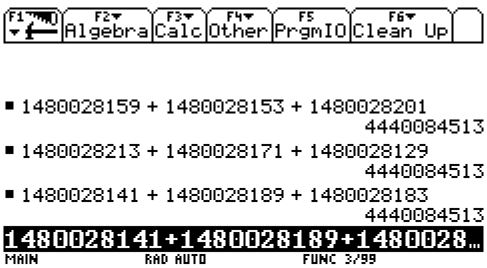


FIGURE 50

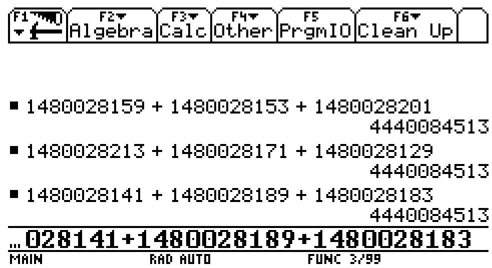


FIGURE 51

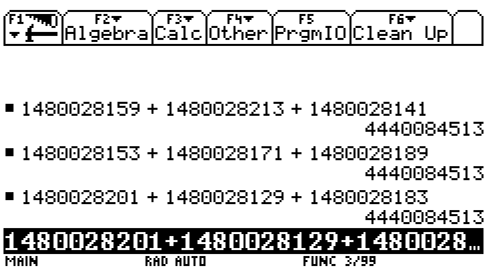


FIGURE 52

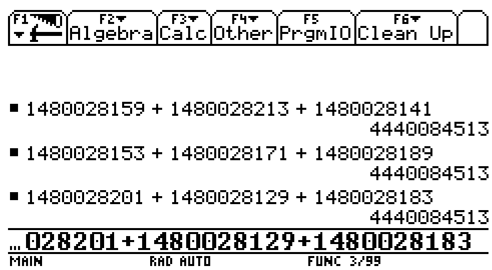


FIGURE 53

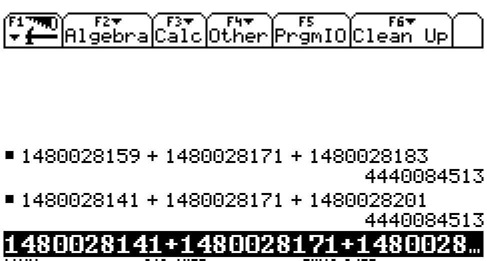


FIGURE 54

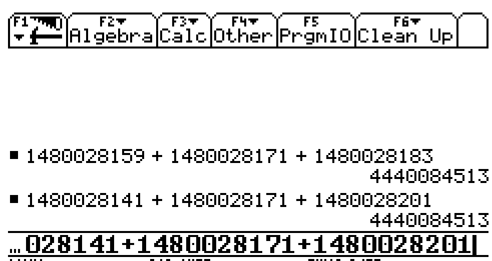


FIGURE 55

The magic sum of 4440084513 is not prime as seen in **FIGURE 56:**

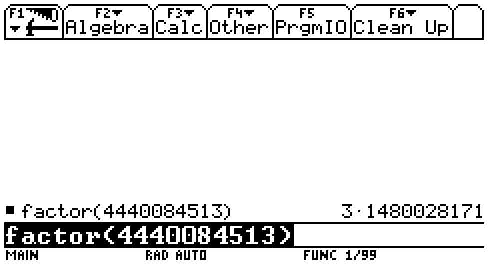


FIGURE 56

IV. In 1975, the largest known pair of twin primes was 140,737,488,353,699 and 140,737,488,353,701.

(a). We prove that the above integers are indeed prime with the aid of the VOYAGE 200 in **FIGURE 57** with the aid of the factor(option and then (b). secure a larger twin prime pair with the aid of the Next Prime program in **FIGURES 58-75:**

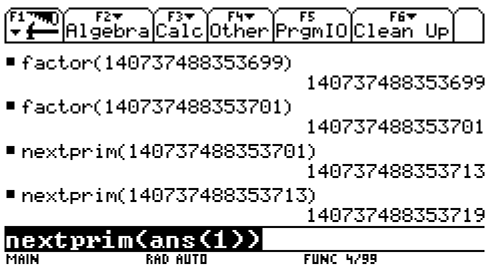


FIGURE 57

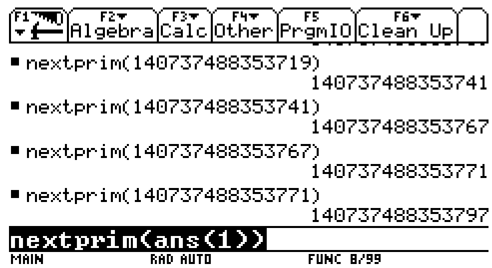


FIGURE 58

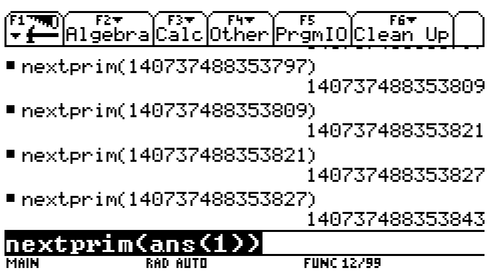


FIGURE 59

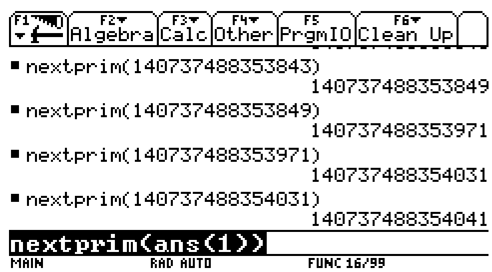


FIGURE 60

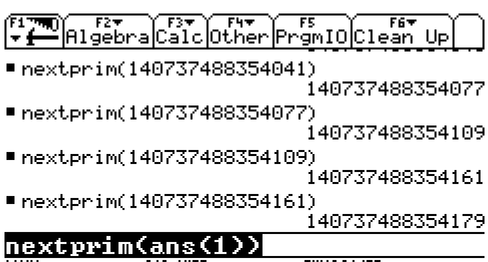


FIGURE 61

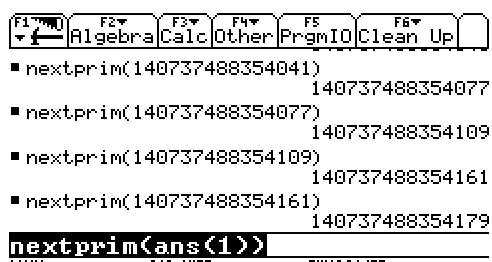


FIGURE 62

```
F1 F2 Algebra | F3 Calc | F4 Other | F5 PrgmIO | F6 Clean Up
■ nextprim(140737488354179)
                140737488354233
■ nextprim(140737488354233)
                140737488354293
■ nextprim(140737488354293)
                140737488354329
■ nextprim(140737488354329)
                140737488354347
nextprim(ans(1))
MAIN    RAD AUTO    FUNC 24/99
```

FIGURE 63

```
F1 F2 Algebra | F3 Calc | F4 Other | F5 PrgmIO | F6 Clean Up
■ nextprim(140737488354347)
                140737488354373
■ nextprim(140737488354373)
                140737488354409
■ nextprim(140737488354409)
                140737488354413
■ nextprim(140737488354413)
                140737488354431
nextprim(ans(1))
MAIN    RAD AUTO    FUNC 28/99
```

FIGURE 64

```
F1 F2 Algebra | F3 Calc | F4 Other | F5 PrgmIO | F6 Clean Up
■ nextprim(140737488354431)
                140737488354511
■ nextprim(140737488354511)
                140737488354557
■ nextprim(140737488354557)
                140737488354613
■ nextprim(140737488354613)
                140737488354679
nextprim(ans(1))
MAIN    RAD AUTO    FUNC 32/99
```

FIGURE 65

```
F1 F2 Algebra | F3 Calc | F4 Other | F5 PrgmIO | F6 Clean Up
■ nextprim(140737488354679)
                140737488354709
■ nextprim(140737488354709)
                140737488354787
■ nextprim(140737488354787)
                140737488354893
■ nextprim(140737488354893)
                140737488354989
nextprim(ans(1))
MAIN    RAD AUTO    FUNC 36/99
```

FIGURE 66

```
F1 F2 Algebra | F3 Calc | F4 Other | F5 PrgmIO | F6 Clean Up
■ nextprim(140737488354989)
                140737488355031
■ nextprim(140737488355031)
                140737488355049
■ nextprim(140737488355049)
                140737488355181
■ nextprim(140737488355181)
                140737488355201
nextprim(ans(1))
MAIN    RAD AUTO    FUNC 40/99
```

FIGURE 67

```
F1 F2 Algebra | F3 Calc | F4 Other | F5 PrgmIO | F6 Clean Up
■ nextprim(140737488355201)
                140737488355213
■ nextprim(140737488355213)
                140737488355333
■ nextprim(140737488355333)
                140737488355337
■ nextprim(140737488355337)
                140737488355369
nextprim(ans(1))
MAIN    RAD AUTO    FUNC 44/99
```

FIGURE 68

```
F1 F2 Algebra | F3 Calc | F4 Other | F5 PrgmIO | F6 Clean Up
■ nextprim(140737488355369)
                140737488355393
■ nextprim(140737488355393)
                140737488355441
■ nextprim(140737488355441)
                140737488355469
■ nextprim(140737488355469)
                140737488355483
nextprim(ans(1))
MAIN    RAD AUTO    FUNC 48/99
```

FIGURE 69

```
F1 F2 Algebra | F3 Calc | F4 Other | F5 PrgmIO | F6 Clean Up
■ nextprim(140737488355369)
                140737488355393
■ nextprim(140737488355393)
                140737488355441
■ nextprim(140737488355441)
                140737488355469
■ nextprim(140737488355469)
                140737488355483
nextprim(ans(1))
MAIN    RAD AUTO    FUNC 48/99
```

FIGURE 70

```
F1 F2 Algebra | F3 Calc | F4 Other | F5 PrgmIO | F6 Clean Up
■ nextprim(140737488355507)
                140737488355601
■ nextprim(140737488355601)
                140737488355609
■ nextprim(140737488355609)
                140737488355781
■ nextprim(140737488355781)
                140737488355819
nextprim(ans(1))
MAIN    RAD AUTO    FUNC 53/99
```

FIGURE 71

```
F1 F2 Algebra | F3 Calc | F4 Other | F5 PrgmIO | F6 Clean Up
■ nextprim(140737488355819)
                140737488355879
■ nextprim(140737488355879)
                140737488355939
■ nextprim(140737488355939)
                140737488355949
■ nextprim(140737488355949)
                140737488356009
nextprim(ans(1))
MAIN    RAD AUTO    FUNC 57/99
```

FIGURE 72

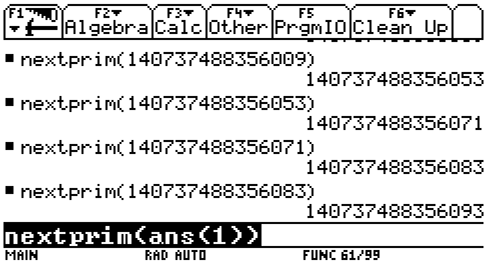


FIGURE 73

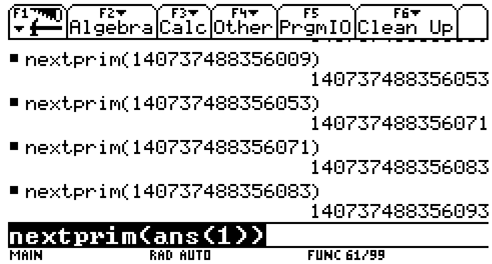


FIGURE 74

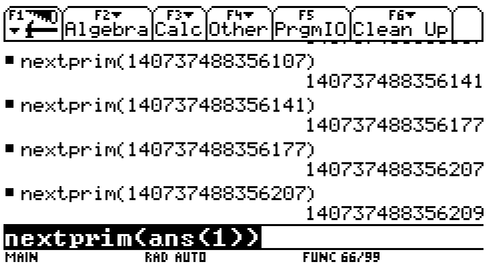


FIGURE 75

We observe after proceeding in excess of five-dozen primes, a larger pair of twin primes is obtained; namely 140,737,488,356,207, and 140,737,488,356,209. It is unknown if the number of twin prime pairs is finite or infinite.

V. A Fun Activity with the Fibonacci sequence

We generate the Fibonacci sequence on the HOME SCREEN. First recall the famous *Fibonacci sequence* is recursively defined as follows:

Define $F_1 = F_2 = 1 \, and \; F_n = F_{n-2} + F_{n-1} \; for \; n \geq 3.$ Here $F_n =$ the nth term of the Fibonacci sequence. We use the VOYAGE 200 to generate the initial forty outputs in the Fibonacci sequence. See **FIGURES 77-82:**

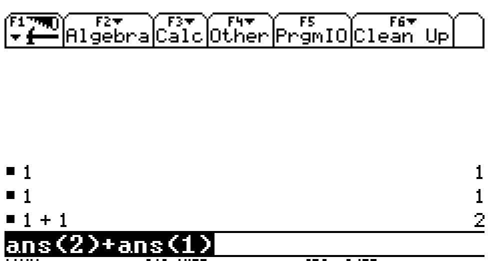


FIGURE 76

In **FIGURE 76,** on The HOME SCREEN, we entered the initial two terms to start the recursion which are both 1 and then used the command $ans(2) + ans(1)$ followed by ENTER. This will furnish the sum of the next to the last answer on the HOME SCREEN followed by the last answer on the HOME SCREEN. Keep pressing ENTER to generate new terms of this sequence. See **FIGURES 77-82:**

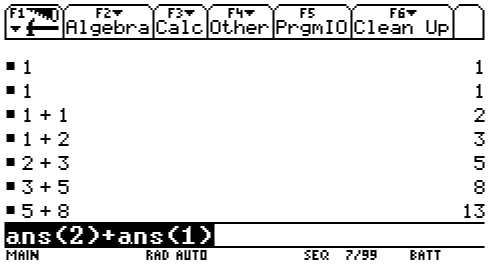


FIGURE 77

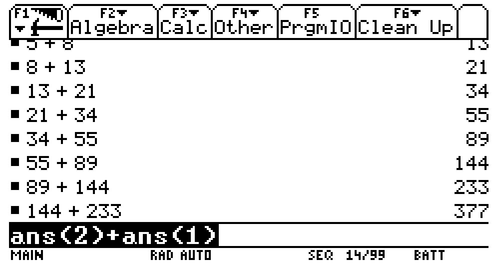


FIGURE 78

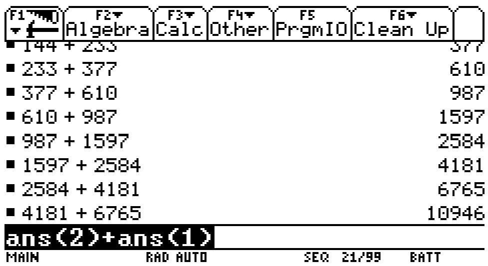


FIGURE 79

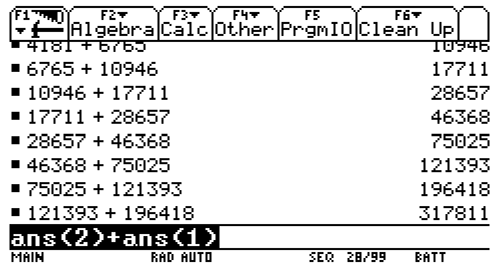


FIGURE 80

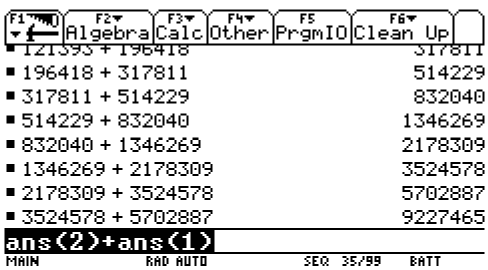


FIGURE 81

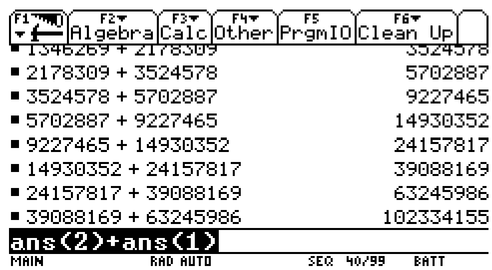


FIGURE 82

If one reads this data, they see two numbers on the bottom right; for example in **FIGURE 71**, one sees 28/99. The 28th term is the last answer in **FIGURE 80** and is 317811. There are 99 possible answers retained by the calculator. One can adjust this last number. From the HOME SCREEN, use the keystrokes F1 9: Format (see **FIGURE 83**) and press ENTER. You will see what is called History Pairs and use the right arrow cursor to see the choices, which indicate the number of answers one can recover from the HOME SCREEN (see **FIGURES 84-85**). The factory setting for the History Pairs is 30.

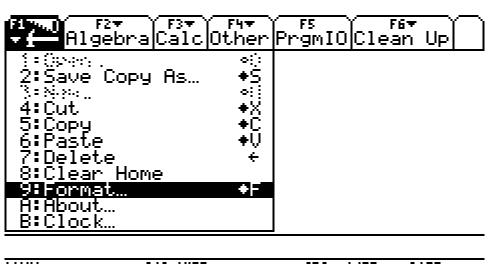


FIGURE 83

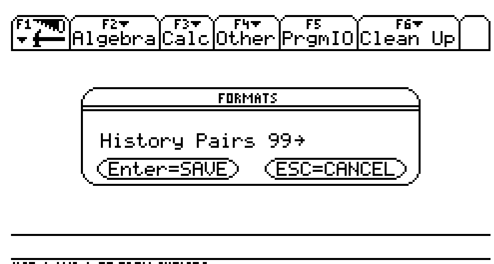


FIGURE 84

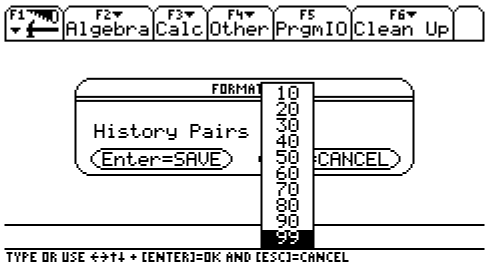


FIGURE 85

Based on the data in **FIGURES 77-82,** we conjecture that every fourth Fibonacci integer is divisible by three.

$$\left\{\begin{array}{l} F_4 = 3, F_8 = 21, F_{12} = 144, F_{16} = 987, F_{20} = 6765 \\ F_5 = 5, F_{10} = 55, F_{15} = 610, F_{20} = 6765, F_{25} = 75025 \\ F_8 = 21, F_{16} = 987, F_{24} = 46368, F_{32} = 2178309, F_{40} = 102334155 \end{array}\right\}.$$

Proceeding to SEQUENCE GRAPHING (use the keystrokes MODE followed by the right arrow cursor to option 4: SEQUENCE followed by ENTER), we see an SEQ at the bottom of the HOME SCREEN. See **FIGURES 86-87:**

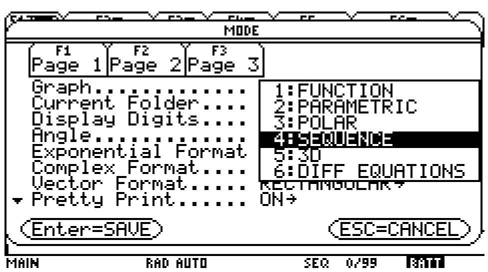


FIGURE 86

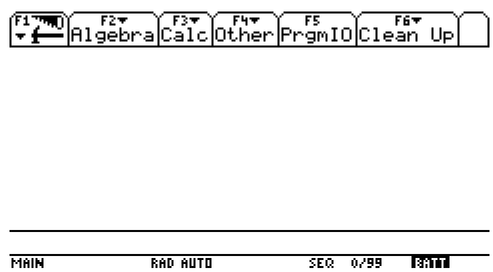


FIGURE 87

Next proceed to the Y= EDITOR and input the following as in **FIGURE 79** with the Standard Viewing Window, Graph, Table Setup, and a portion of the TABLE in **FIGURES 88-94:**

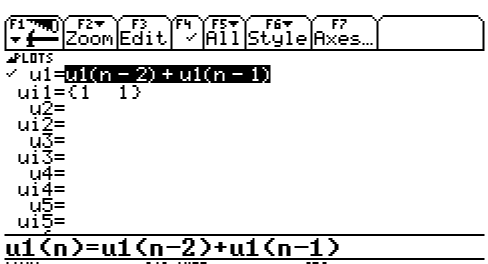


FIGURE 88

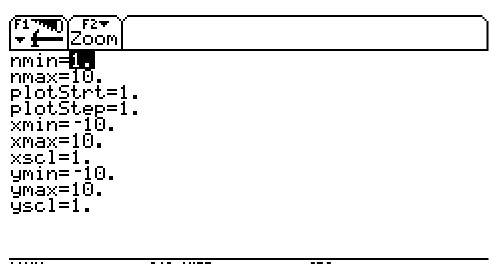


FIGURE 89

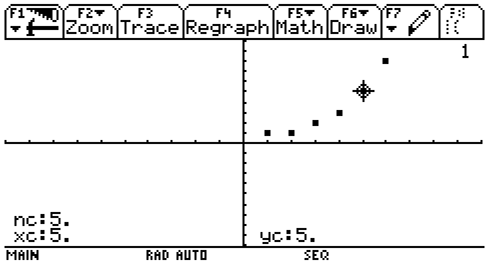


FIGURE 90

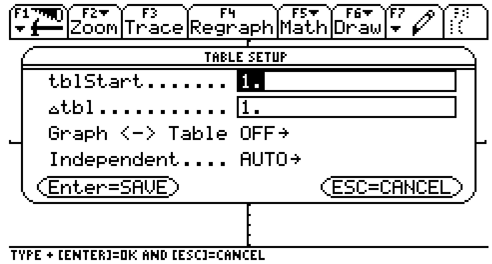


FIGURE 91

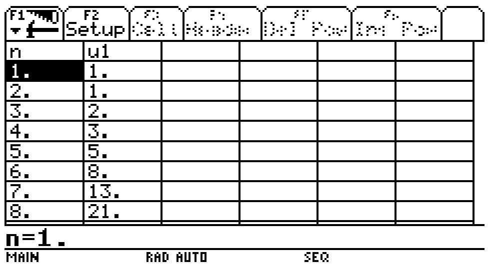


FIGURE 92

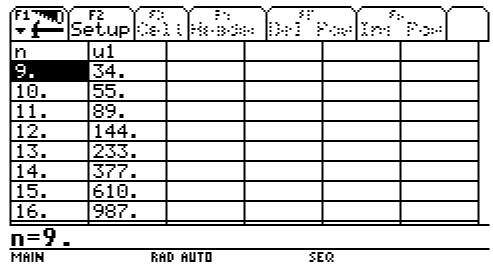


FIGURE 93

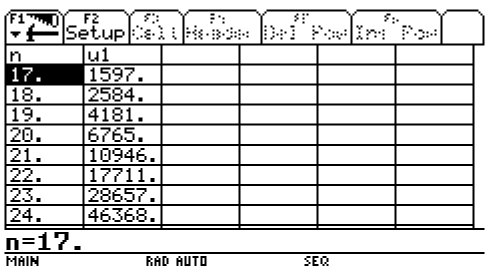


FIGURE 94

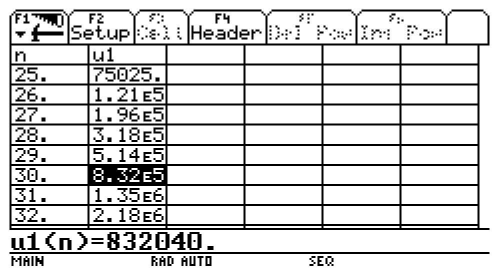


FIGURE 95

<u>Some Comments on the above screen captures</u>:

1. **In FIGURE 87,** note that the recursion rule is provided on the line headed by $u1$ while the line headed by $ui1$ records the initial two terms of the sequence, the second followed by the first. There is no comma between the two 1's in Pretty Print although one separates the two initial 1's with a comma on the entry line. See **FIGURE 96:**

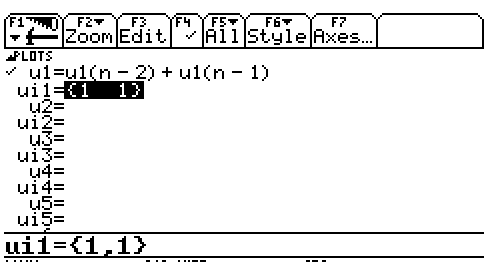


FIGURE 96

2. Note from **FIGURE 92** that 5 is the fifth term of the Fibonacci sequence.
3. Since a sequence is a function whose domain is the set of positive integers, the Tbl Start begins at 1 in **FIGURE 91.**

4. Only five figures are possible in any cell. Thus all terms of the Fibonacci sequence after the twenty-fifth are expressed in scientific notation. If one places their cursor on the output value, however, the exact value is determined as in **FIGURE 95** where the thirtieth term is given exactly as 832040.

Thus if one considers the famous Fibonacci sequence or any Fibonacci-like sequence (that is a sequence whose first two terms can be anything one pleases but each term thereafter follows the recursion rule in the Fibonacci sequence), form the sum of any six consecutive terms and divide this sum by four. We do this for three separate sets and form a conjecture. The results are tabulated in the following TABLE:

SUM OF SIX CONSECUTIVE FIBONACCI NUMBERS:	SUM OF THE MEMBERS OF THE SET:	QUOTIENT WHEN THE SUM IS DIVIDED BY 4:
$\{2,3,5,8,13,21\}$	52	13…FIFTH TERM
$\{1,1,2,3,5,8\}$	20	5…FIFTH TERM
$\{55,89,144,233,377,610\}$	1508	377…FIFTH TERM

<u>CONJECTURE:</u> The sum of any six consecutive Fibonacci numbers is divisible by 4 and the quotient will always be the fifth term in the sequence.

<u>Proof:</u> Consider the initial two terms of the Fibonacci sequence to be $x\ and\ y.$ The six consecutive terms of the sequence are as follows:
$\{x, y, x+y, x+2\cdot y, 2\cdot x+3\cdot y, 3\cdot x+5\cdot y\}.$ We employ the VOYAGE 200 to form the sum and divide the resulting sum by 4. See **FIGURE 97:**

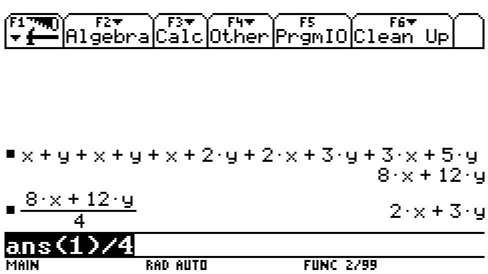


FIGURE 97

Let us next form the sum of any ten consecutive integers and divide this sum by 11. View this for three separate sets and form a conjecture. The results are tabulated below:

View this for three separate sets and form a conjecture. The results are tabulated below: SUM OF TEN CONSECUTIVE FIBONACCI NUMBERS:	SUM OF THE MEMBERS OF THE SET:	QUOTIENT WHEN THE SUM IS DIVIDED BY 11:
$\{2,3,5,8,13,21,34,55,89,144\}$	374	34…SEVENTH TERM

$\{1,1,2,3,5,8,13,21,34,55\}$	143	13…SEVENTH TERM
$\{55,89,144,233,377,610,987,1597,2584,4181\}$	10857	987…SEVENTH TERM

<u>CONJECTURE:</u> The sum of any ten consecutive Fibonacci numbers is divisible by 11 and the quotient will always be the seventh term in the sequence.

<u>Proof:</u> Consider the initial two terms of the Fibonacci sequence to be $x\,and\,y.$ The ten consecutive terms of the sequence are as follows:

$$\{x, y, x+y, x+2\cdot y, 2\cdot x+3\cdot y, 3\cdot x+5\cdot y, 5\cdot x+8\cdot y, 8\cdot x+13\cdot y, 13\cdot x+21\cdot y, 21\cdot x+34\cdot y\}$$

Let us employ the TI-89 to form the sum and divide the resulting sum by 11. See **FIGURES 98-101:**

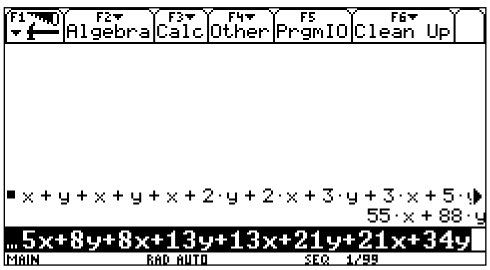


FIGURE 98

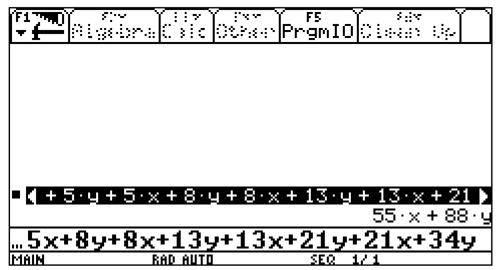


FIGURE 99

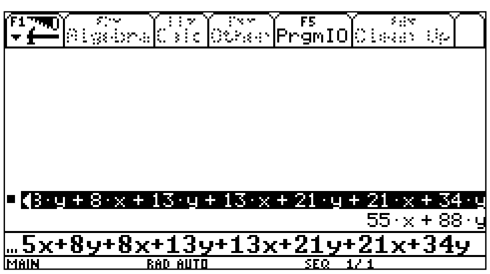


FIGURE 100

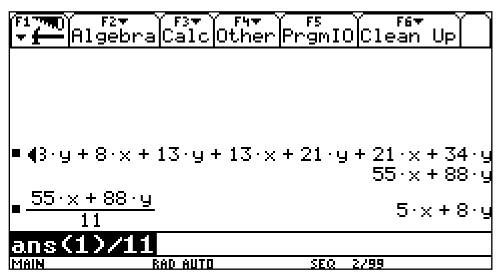


FIGURE 101

Notice $5\cdot x+8\cdot y$ is the seventh term in the sequence which is a neat Fibonacci number trick.

Let us next form the sum of any fourteen consecutive integers and divide this sum by 29 for three separate sets and form a conjecture. The results are tabulated below:

SUM OF FOURTEEN CONSECUTIVE FIBONACCI NUMBERS:	SUM OF THE MEMBERS OF THE SET:	QUOTIENT WHEN THE SUM IS DIVIDED BY 29:
$\{2,3,5,8,13,21,34,55,89,144, 233, 377, 610, 987\}$	2581	89…NINTH TERM

$\{1,1,2,3,5,8,13,21,34,55,\ 89,\ 144,\ 233,\ 377\}$	986	34…NINTH TERM
$\left\{\begin{matrix}55,89,144,233,377,610,987,1597,2584,\\ 4181,6765,\ 10946,\ 17711,\ 28657\end{matrix}\right\}$	74936	2584…NINTH TERM

<u>CONJECTURE:</u> The sum of any fourteen consecutive Fibonacci numbers is divisible by 29 and the quotient will always be the ninth term in the sequence.

<u>Proof:</u> Consider the initial two terms of the Fibonacci sequence to be $x\, and\ y.$ The fourteen consecutive terms of the sequence are as follows:

$$\left\{\begin{matrix}x,y,x+y,x+2\cdot y,2\cdot x+3\cdot y,3\cdot x+5\cdot y,5\cdot x+8\cdot y,8\cdot x+13\cdot y,13\cdot x+21\cdot y,21\cdot x+34\cdot y,\\ 34\cdot x+55\cdot y,\ 55\cdot x+89\cdot y,\ 89\cdot x+144\cdot y,144\cdot x+233\cdot y\end{matrix}\right\}$$

The VOYAGE 200 is used to form the sum and divide the total by 29. See **FIGURES 102-106:**

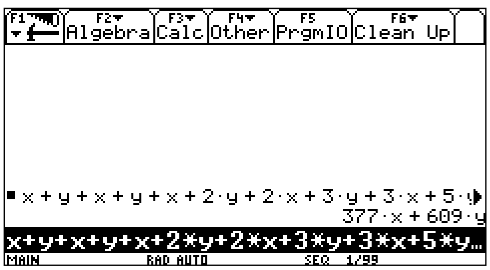


FIGURE 102

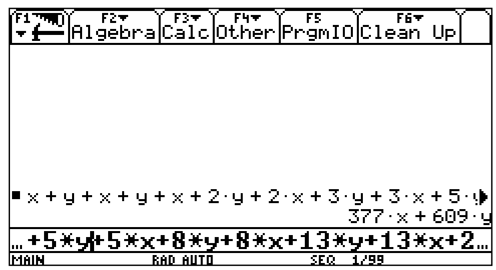


FIGURE 103

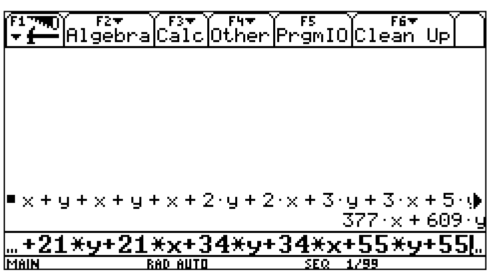


FIGURE 104

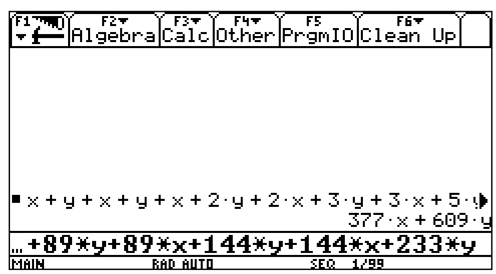


FIGURE 105

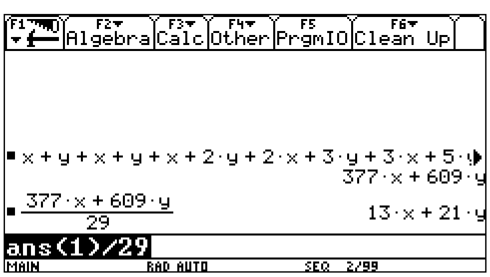


FIGURE 106

VI. Geometry and the Fibonacci Sequence.

In this activity, we next take any four consecutive Fibonacci numbers. Form the product of the first and fourth terms of the sequence. Next take twice the product of the second and terms. Finally take the sum of the squares of the second and third terms. Observe the relationship to the Pythagorean Theorem in plane geometry. We gather some empirical evidence via the following three examples:

Example 1: Consider the set of four consecutive Fibonacci numbers $\{3,5,8,13\}$. Observe the truth of the following with the aid of the VOYAGE 200. See **FIGURE 107:**

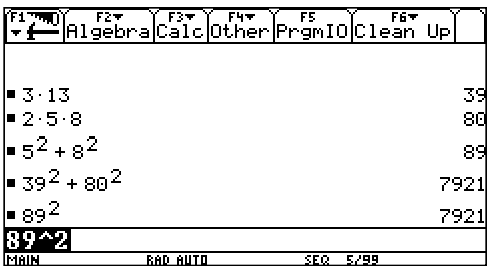


FIGURE 107

Observe that the primitive Pythagorean Triple (39, 80, 89) is formed.

Example 2: We next consider the sequence of four consecutive Fibonacci numbers $\{8,13,21,34\}$. We observe the truth of the following computations furnished by the VOYAGE 200. See **FIGURE 108:**

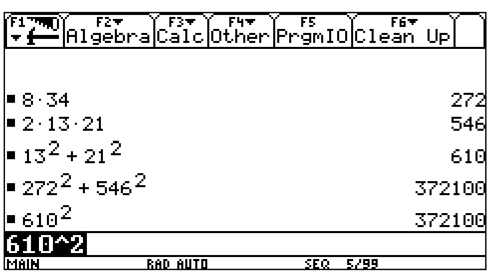


FIGURE 108

The Pythagorean Triple (272, 546, 610) (albeit not primitive; for 2 is a common factor among each of the components) is formed. The associated primitive Pythagorean Triple is (136, 273, 305).

Example 3: Consider the sequence of four consecutive Fibonacci numbers $\{13,21,34,55\}$. See **FIGURE 109** for the relevant computations.

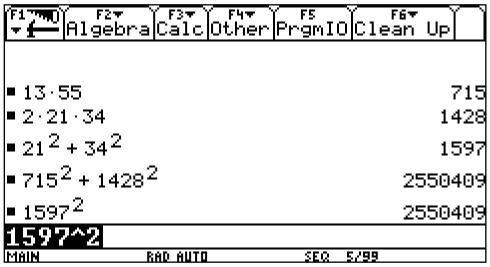


FIGURE 109

The primitive Pythagorean triple (715, 1428, 1597) is formed.
Note that the hypotenuses of each of the right triangles formed are Fibonacci numbers. (89, 610, 1597).

Based on the observations in the three examples, one suspects that a Pythagorean triple is always formed and this is indeed the case. We justify our conjecture with the aid of the VOYAGE 200:

Suppose $\{x, y, x+y, x+2\cdot y\}$ represent any four consecutive terms of the Fibonacci (or Fibonacci-like sequence). We view our inputs and outputs in **FIGURE 111** using the expand (command (See **FIGURE 110)** from the Algebra menu on the HOME SCREEN:

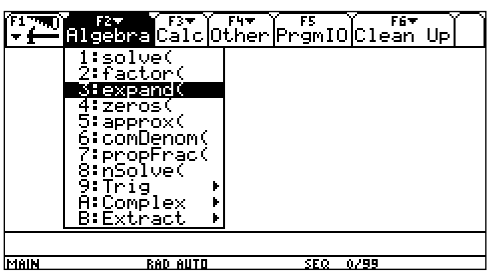


FIGURE 110

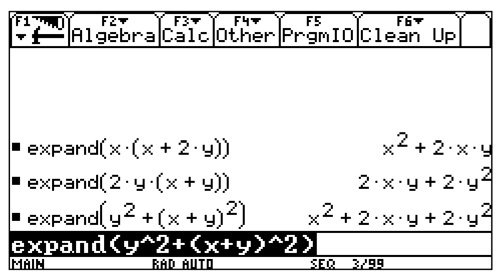


FIGURE 111

To show that $\left(x^2+2\cdot x\cdot y, 2\cdot x\cdot y+2\cdot y^2, x^2+2\cdot x\cdot y+2\cdot y^2\right)$ forms a Pythagorean Triple, see **FIGURES 112-114** for our inputs and outputs:

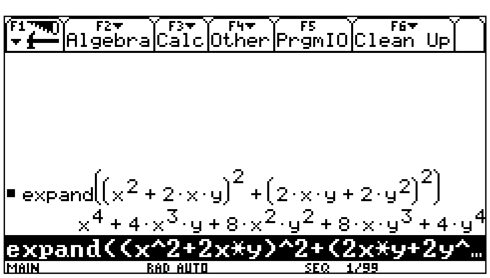


FIGURE 112

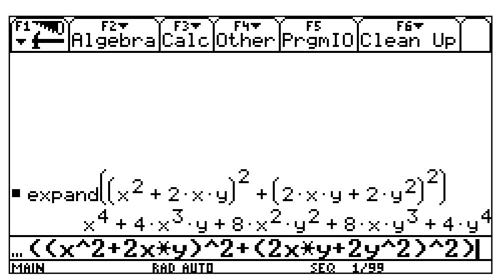


FIGURE 113

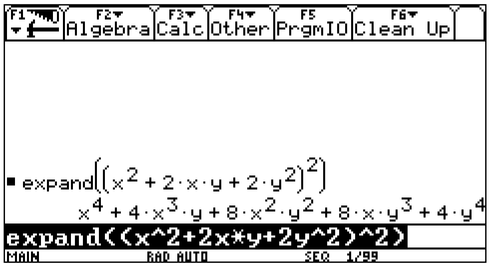


FIGURE 114

VII. If we add two unit fractions with consecutive even denominators, starting with $\frac{1}{2}$, we obtain the following for fifteen iterations in **FIGURES 115-117:**

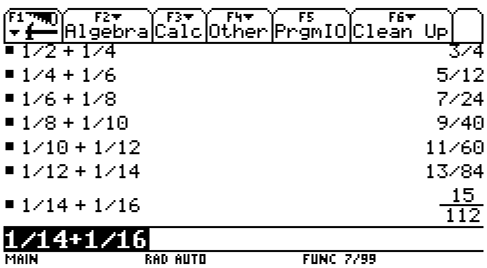


FIGURE 115

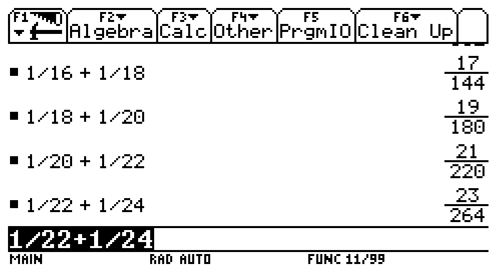


FIGURE 116

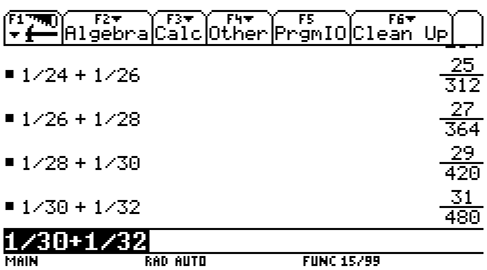


FIGURE 117

Similarly if we add two unit fractions with consecutive odd denominators starting with 1/3, we obtain the following for fifteen iterations in **FIGURES 118-120:**

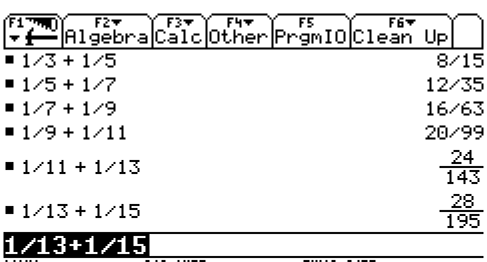


FIGURE 118

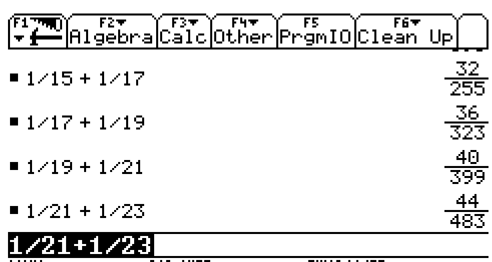


FIGURE 119

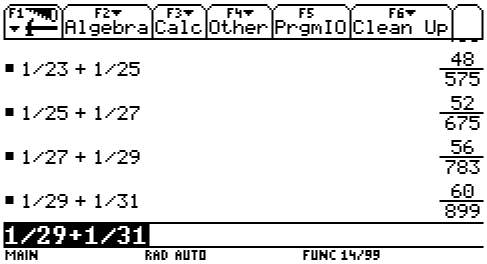


FIGURE 120

Consider the numerators and denominators of the sums obtained in **FIGURES 115-120.** We find that all form the legs of Primitive Pythagorean Triangles. In **FIGURES 121-130,** we show that the sum of the squares of the numerators and denominators of each of these fractions is a perfect square and hence a Pythagorean Triple is formed. Moreover, since $(a,b,c)=1$ in the sense that there are no common integer factors other than 1 among the components, the triples are classified as Primitive Pythagorean triples.

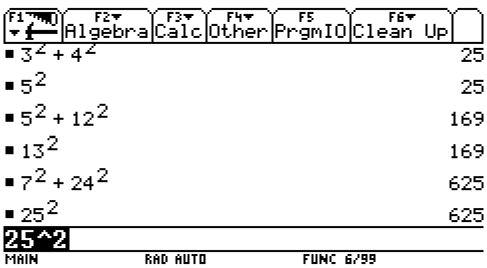


FIGURE 121

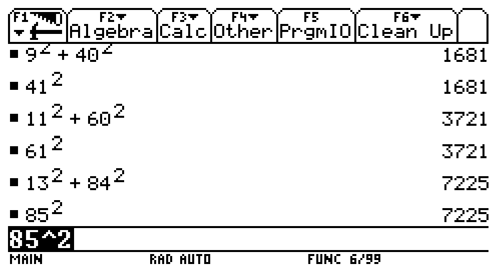


FIGURE 122

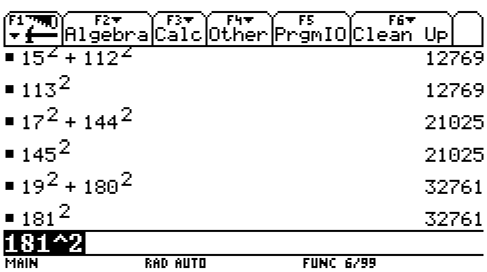


FIGURE 123

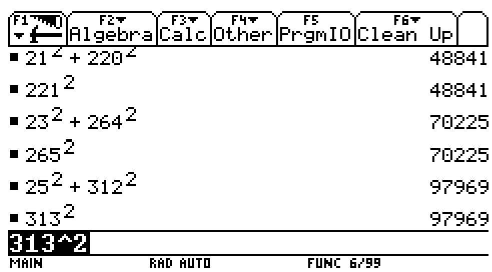


FIGURE 124

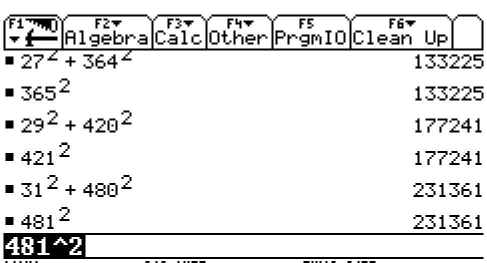


FIGURE 125

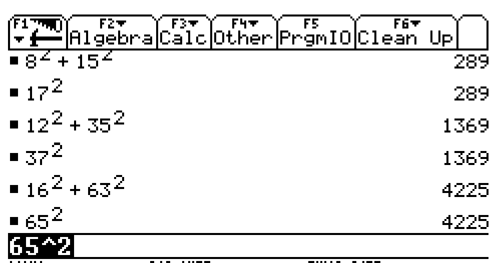


FIGURE 126

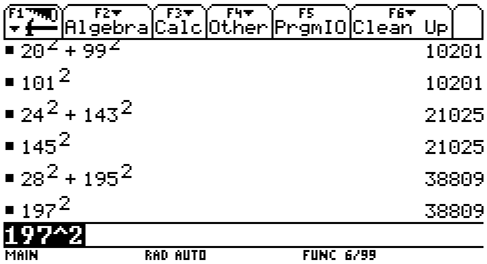


FIGURE 127

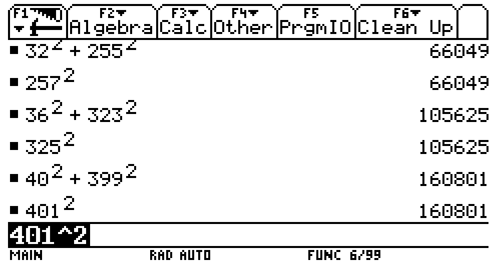


FIGURE 128

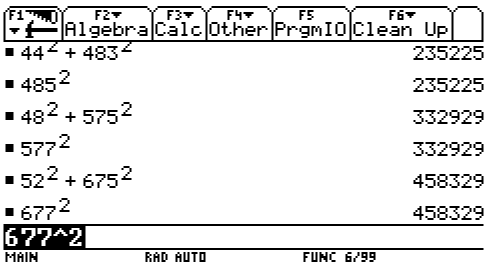


FIGURE 129

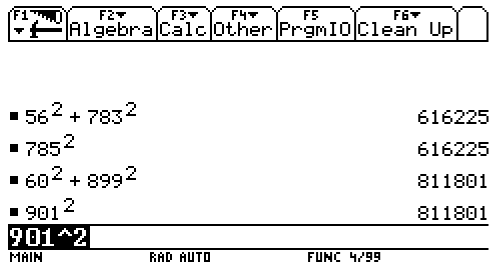


FIGURE 130

Are these results always true and are all Primitive Pythagorean Triples obtained in this manner? The answers are YES and NO respectively. The latter question can be resolved by noting that the PPT (77, 36, 85) is not generated by this process. See **FIGURE 131.** Also see **FIGURES 116-119** for a general proof considering the cases of the sum of consecutive even unit fractions and the sum of consecutive odd unit fractions separately with **FIGURE 115** displaying the relevant Algebra Menu F2 on the TI-89/VOYAGE 200.

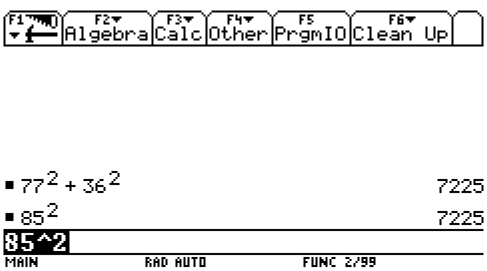


FIGURE 131

One can show that the following is true in general. If we consider that any even integer is of the form $2\cdot m$ for some integer m and any odd integer is of the form $2\cdot n+1$ for some integer $n,$ then we have the following for the sum of two unit fractions with consecutive even denominators:

$\frac{1}{2\cdot m}+\frac{1}{2\cdot m+2}=\frac{2\cdot m+1}{2\cdot m^2+2\cdot m}$. Taking the sum of the squares of the numerator and denominator of this fraction, we note

$\left(2\cdot m+1\right)^2+\left(2\cdot m^2+2\cdot m\right)^2=4\cdot m^4+8\cdot m^3+8\cdot m^2+4\cdot m+1=\left(2\cdot m^2+2\cdot m+1\right)^2$. Hence one obtains the Primitive Pythagorean triple $\left(2\cdot m+1,\ 2\cdot m^2+2\cdot m,\ 2\cdot m^2+2\cdot m+1\right)$.

For the sum of two unit fractions with consecutive odd denominators, we observe the following:

$\frac{1}{2\cdot n+1}+\frac{1}{2\cdot n+3}=\frac{4\cdot n+4}{4\cdot n^2+8\cdot n+3}$. Taking the sum of the squares of the numerator and denominator of this fraction, we note that

$(4\cdot n+4)^2+(4\cdot n^2+8\cdot n+3)^2=16\cdot n^4+64\cdot n^3+104\cdot n^2+80\cdot n+25=(4\cdot n^2+8\cdot n+5)^2$.

Thus one obtains the Primitive Pythagorean Triple

$(4\cdot n+4,\ 4\cdot n^2+8\cdot n+3,\ 4\cdot n^2+8\cdot n+5)$. See **FIGURES 132-136:**

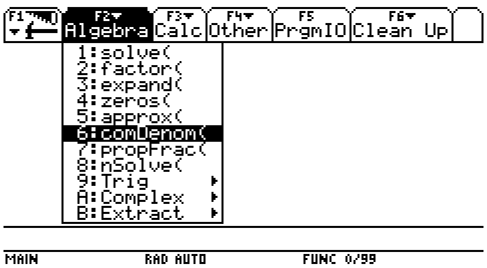


FIGURE 132

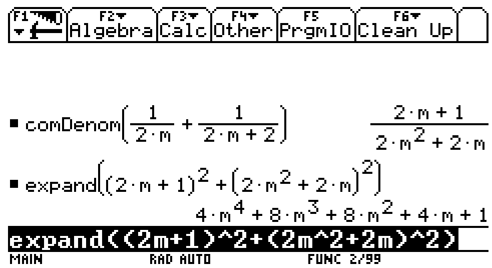


FIGURE 133

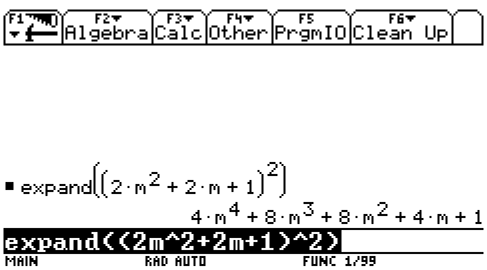


FIGURE 134

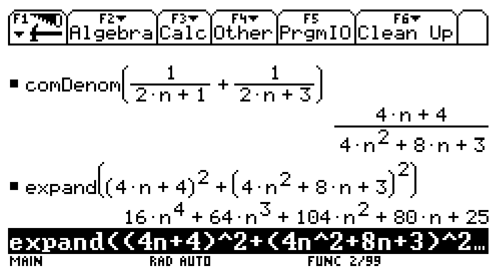


FIGURE 135

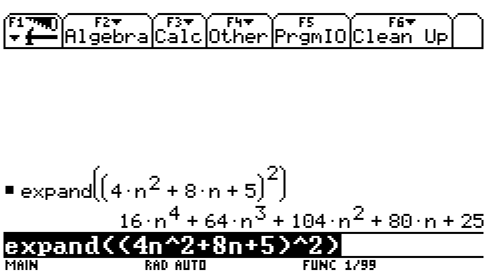


FIGURE 136

VIII. We determine the number of iterations needed to reach 1 for the following positive integers 18, 128, and 25 respectively.

See **FIGURE 137** for the Collatz Program and view **FIGURES 138-140** for the integer 18, **FIGURE 141** for the integer 128 and **FIGURES 142-145** for the integer 25. Here is the PROGRAM:

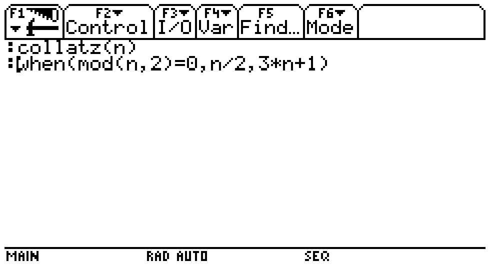


FIGURE 137

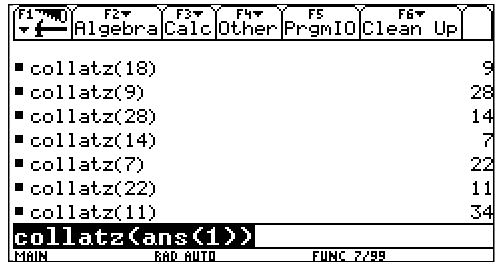


FIGURE 138

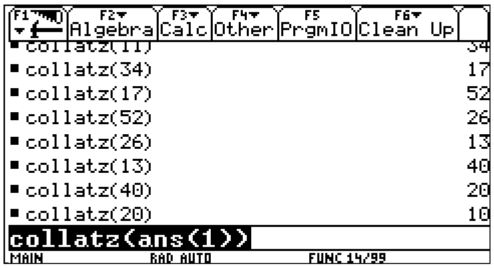


FIGURE 139

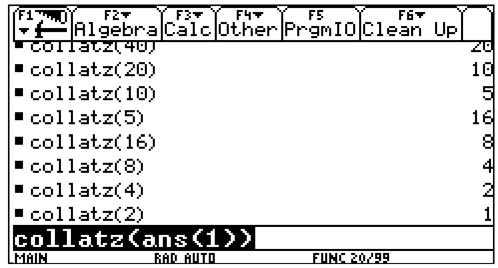


FIGURE 140

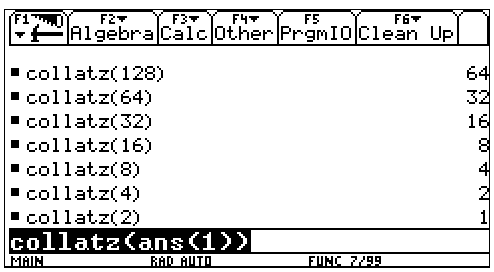


FIGURE 141

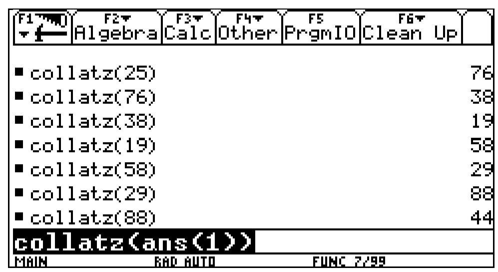


FIGURE 142

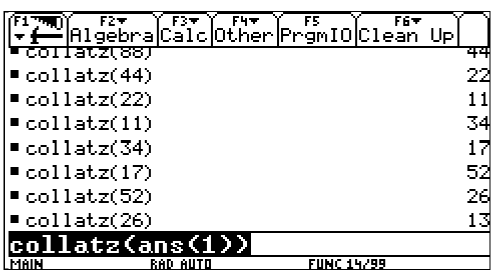


FIGURE 143

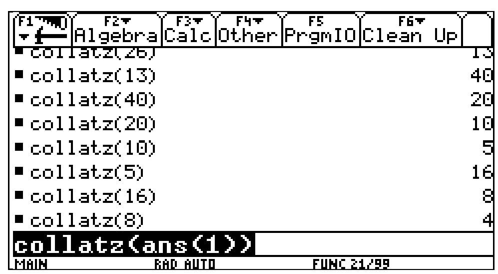


FIGURE 144

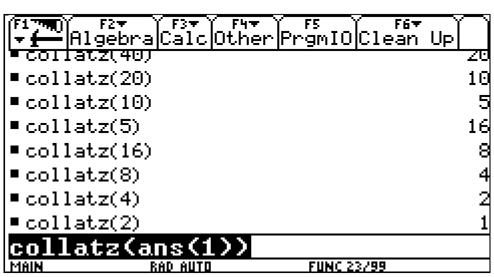


FIGURE 145

Observe that if we do not count the seed value, it takes, in turn, 19, 7, and 23 steps respectively, for the integers 18, 128, and 25 to reach 1.

Alternatively, with another example, let us employ the VOYAGE 200 in determining the number of iterations required for the sequence $\{65, 66, 67\}$ to reach 1. Our inputs and outputs are provided in **FIGURES 146-154** where we are in SEQUENCE MODE:

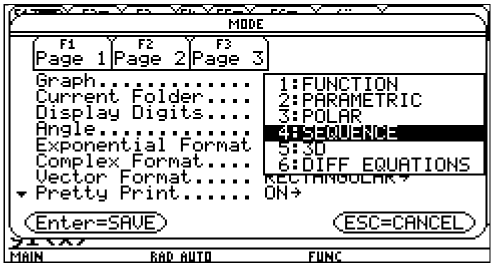


FIGURE 146

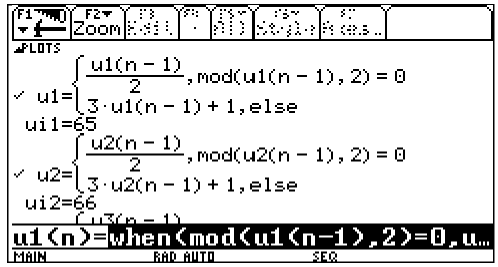


FIGURE 147

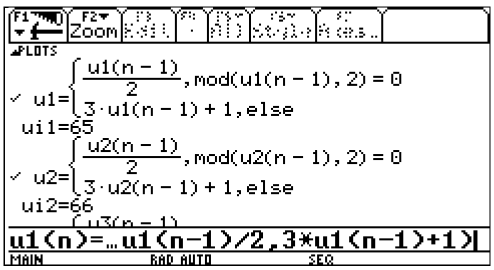


FIGURE 148

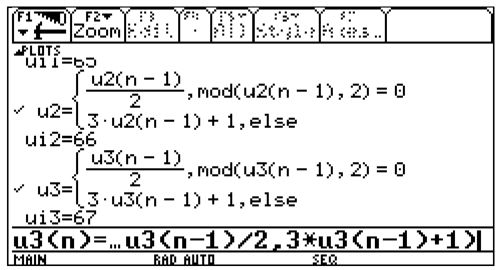


FIGURE 149

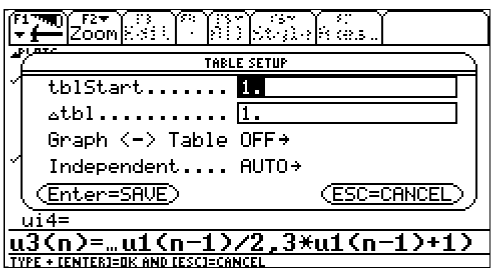
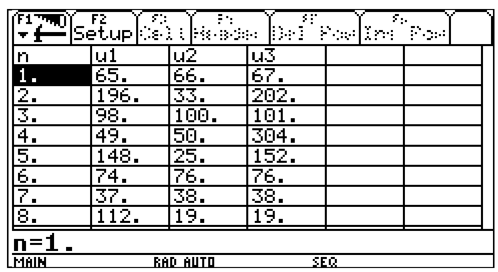


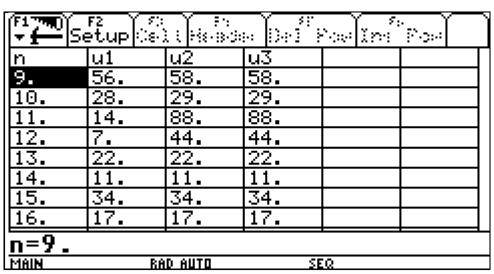
FIGURE 150



n	u1	u2	u3
1.	65.	66.	67.
2.	196.	33.	202.
3.	98.	100.	101.
4.	49.	50.	304.
5.	148.	25.	152.
6.	74.	76.	76.
7.	37.	38.	38.
8.	112.	19.	19.

n=1.

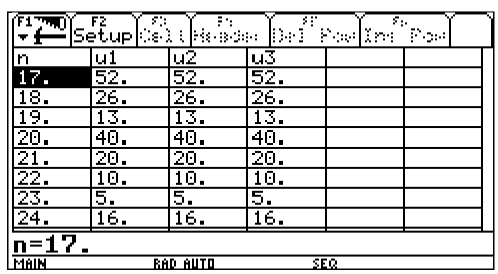
FIGURE 151



n	u1	u2	u3
9.	56.	58.	58.
10.	28.	29.	29.
11.	14.	88.	88.
12.	7.	44.	44.
13.	22.	22.	22.
14.	11.	11.	11.
15.	34.	34.	34.
16.	17.	17.	17.

n=9.

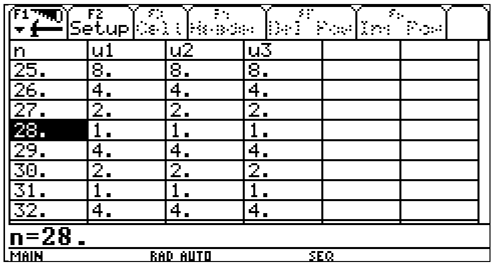
FIGURE 152



n	u1	u2	u3
17.	52.	52.	52.
18.	26.	26.	26.
19.	13.	13.	13.
20.	40.	40.	40.
21.	20.	20.	20.
22.	10.	10.	10.
23.	5.	5.	5.
24.	16.	16.	16.

n=17.

FIGURE 153



n	u1	u2	u3		
25.	8.	8.	8.		
26.	4.	4.	4.		
27.	2.	2.	2.		
28.	1.	1.	1.		
29.	4.	4.	4.		
30.	2.	2.	2.		
31.	1.	1.	1.		
32.	4.	4.	4.		

n=28.

MAIN RAD AUTO SEQ

FIGURE 154

It thus requires 27 steps (not including the initial integers) for the Collatz 3-tuple to reach 1. We are asserting here that 65, 66, and 67 are three consecutive integers for which the Collatz sequence has the same length.

THANK YOU FOR YOUR PARTICIPATION AT THIS WORKSHOP DURING THE AMTNJ 2017 ANNUAL CONFERENCE AT THE NATIONAL CONFERENCE CENTER IN EAST WINDSOR, NJ!