Using Exploration and Technology to Teach Graph Translations

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Opening

- Introduction

- Motivation – learning the “alphabet soup” of graph translations. How do we help students to understand intuitively instead of memorizing?

Equations

- We give students things like: $y = a(x-h)^n + k$

$$y = a\sin(bx+c)+d$$

- Then we give them a similar set of equations for the other trig functions, for log functions, for square root functions, etc.
- After a while, its hard to just memorize everything.

- Our Precalculus text has a chart like this...
- Okay ending point, but not as good for a starting point.

In each case, c represents a positive real number.

To Graph:	Draw the Graph of f and:	Changes in the Equation of $y = f(x)$
Vertical shifts		
$y = f(x) + c$	Raise the graph of f by c units.	c is added to $f(x)$.
$y = f(x) - c$	Lower the graph of f by c units.	c is subtracted from $f(x)$.
Horizontal shifts		
$y = f(x + c)$	Shift the graph of f to the left c units.	x is replaced with $x + c$.
$y = f(x - c)$	Shift the graph of f to the right c units.	x is replaced with $x - c$.
Reflection about the x-axis $y = -f(x)$	Reflect the graph of f about the x-axis.	$f(x)$ is multiplied by -1.
Reflection about the y-axis $y = f(-x)$	Reflect the graph of f about the y-axis.	x is replaced with $-x$.
Vertical stretching or shrinking		
$y = cf(x), c > 1$	Multiply each y-coordinate of $y = f(x)$ by c, vertically stretching the graph of f.	$f(x)$ is multiplied by c, $c > 1$.
$y = cf(x), 0 < c < 1$	Multiply each y-coordinate of $y = f(x)$ by c, vertically shrinking the graph of f.	$f(x)$ is multiplied by c, $0 < c < 1$.
Horizontal stretching or shrinking		
$y = f(cx), c > 1$	Divide each x-coordinate of $y = f(x)$ by c, horizontally shrinking the graph of f.	x is replaced with cx, $c > 1$.
$y = f(cx), 0 < c < 1$	Divide each x-coordinate of $y = f(x)$ by c, horizontally stretching the graph of f.	x is replaced with cx, $0 < c < 1$.

Graphing with Technology

- Graphing Calculator – powerful, compact, portable, expensive. Can be hooked up to a projector with special tools.

- Desmos – best if you have computer access, can be done using a smartphone, free. May be easier to present in class.

- Even if you don't have access to either in class, you can create an exploration assignment before teaching the section and students can use a computer lab, library, or home computer to develop some ideas to share when you teach the lesson the next day.

Order of Operations

- This is a good time to remind students about order of operations.
- The TI graphing calculators are picky about how you enter things. They follow order of operations strictly.

- Entering y = 1/x+3 is not the same as y = 1/(x+3)
- Desmos will give you the fraction bar, but a reminder is still helpful.
- If students don't realize this, they won't make accurate conclusions from your activity.

Notes about these Activities

- As we go through, I’m going to use the word “calculator” as a generic term for technology.

- Depending on your audience, time you have, material to cover, etc., you might do more examples or fewer.
- You might also skip plotting points and go right to the technology.
- Its important that students see one of each type of transformation. Applying it to additional parent functions can be helpful too.
- NOTE: On Desmos, you can get a table of values by going to the “edit” gear on top and then clicking the table icon.

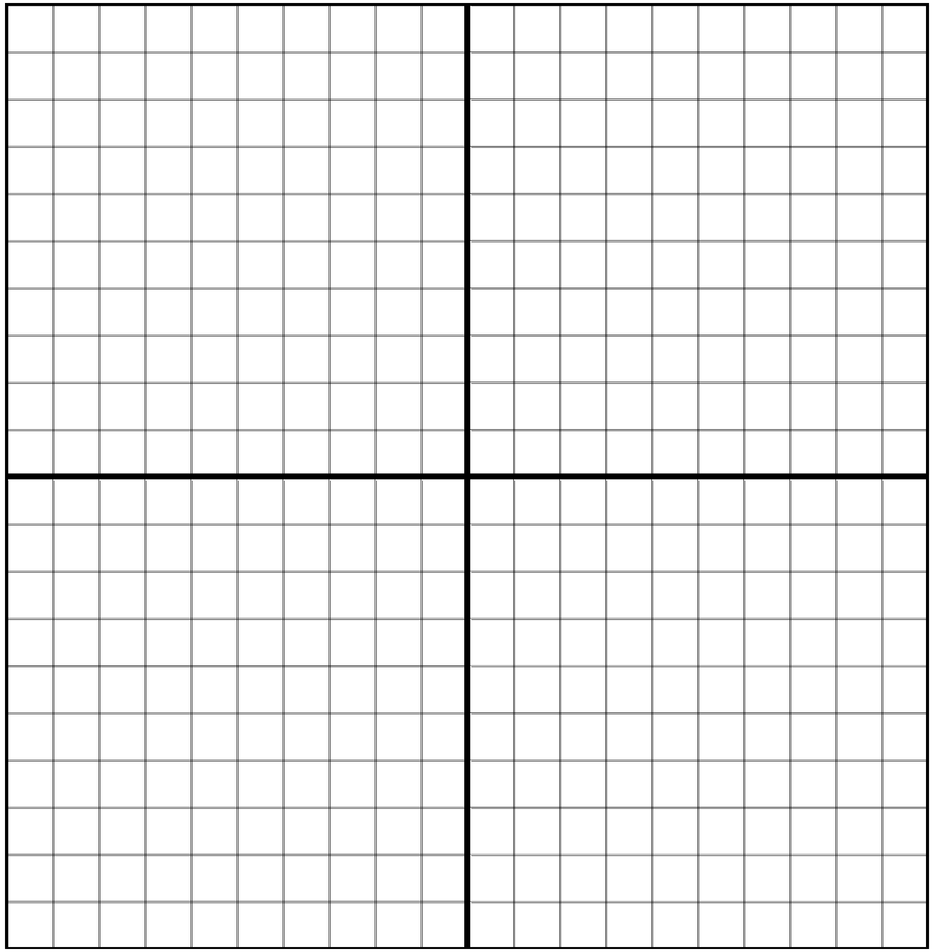
Vertical and Horizontal Translations 1

- Make a table of values and graph:

$$f(x) = x^2$$

$$f(x) = (x-3)^2$$

$$f(x) = x^2 - 3$$



- Then graph using technology.
- Describe what you see. Write something down!

Vertical and Horizontal Translations 2

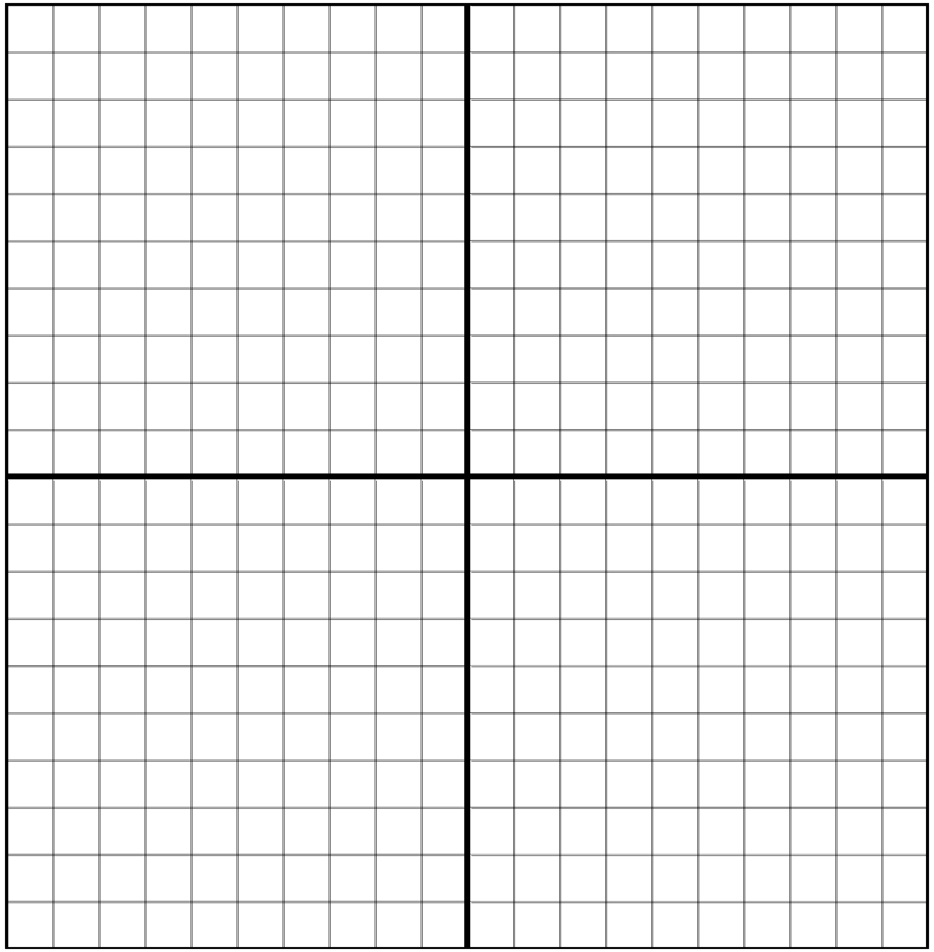
- Make a table of values and graph:

$$f(x) = x^2$$

$$f(x) = (x+3)^2$$

$$f(x) = x^2 + 3$$

- Then graph using technology.
- Describe what you see. Write something down.



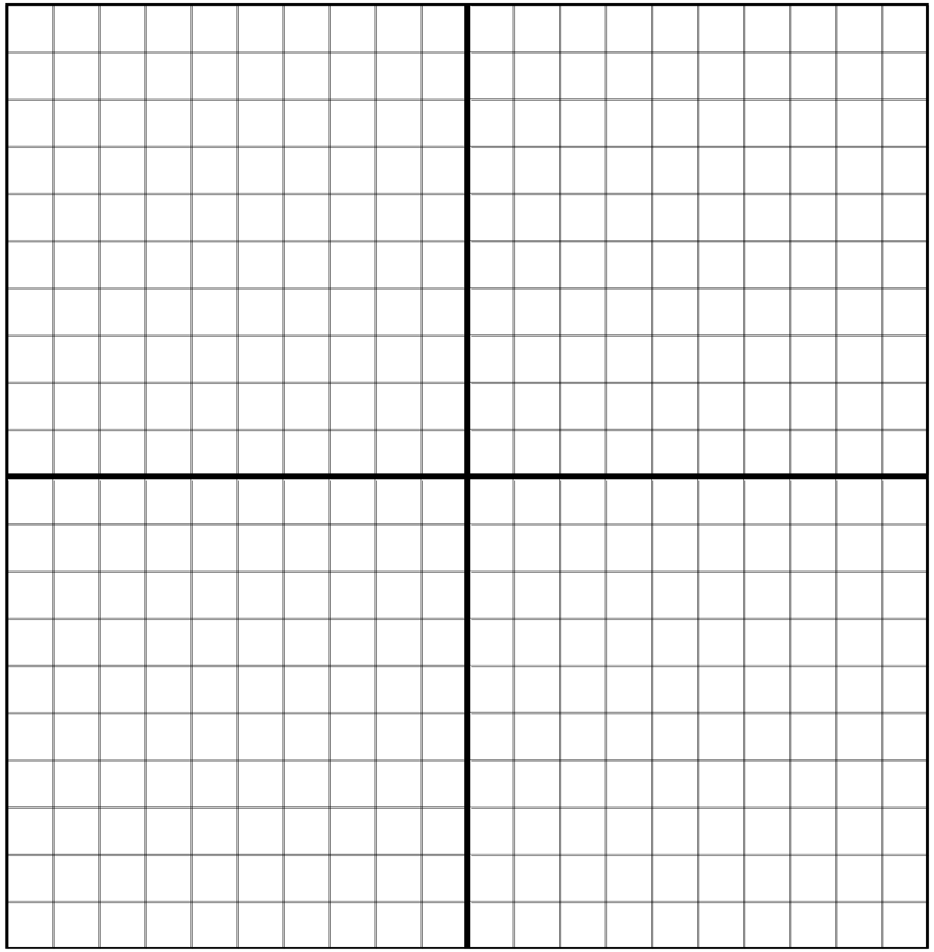
Vertical and Horizontal Translations 3

$$f(x) = \frac{1}{x}$$

$$f(x) = \frac{1}{x+3}$$

$$f(x) = \frac{1}{x} + 3$$

- Graph. What do you notice? Talk about the asymptotes.



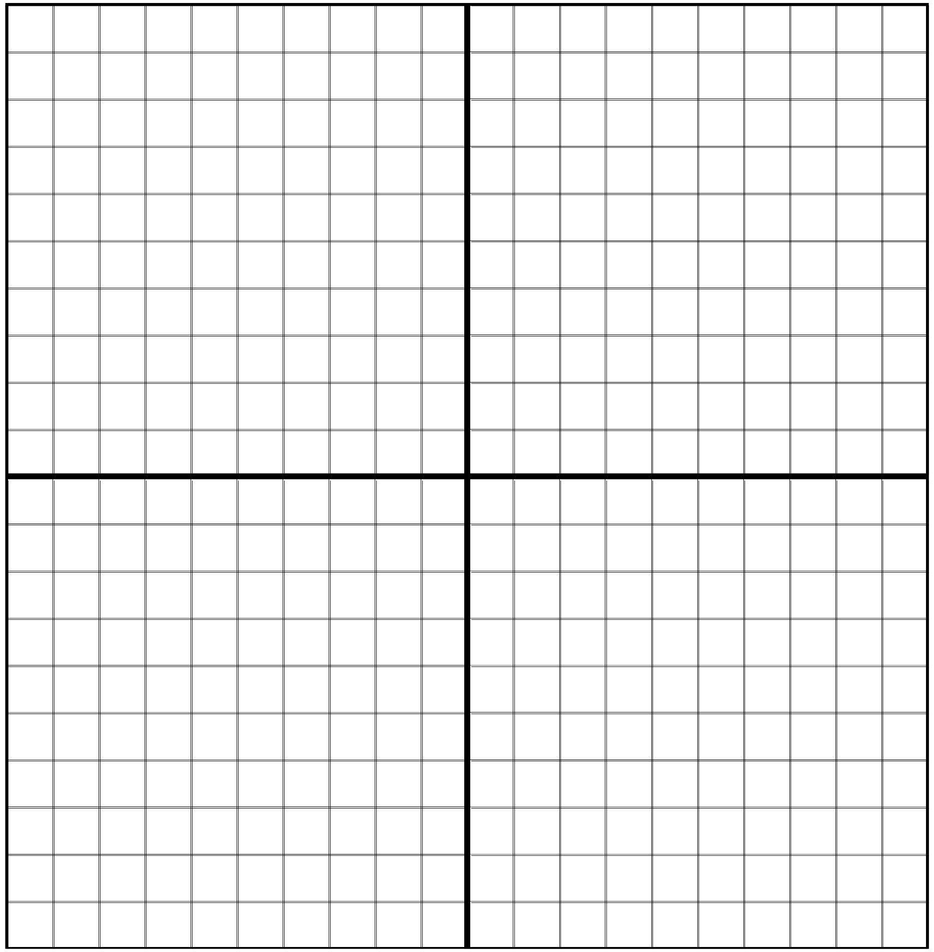
Vertical and Horizontal Translations 4

$$f(x) = \ln(x)$$

$$f(x) = \ln(x-4)$$

$$f(x) = \ln(x) - 4$$

- Graph. What do you notice? Talk about the asymptotes.



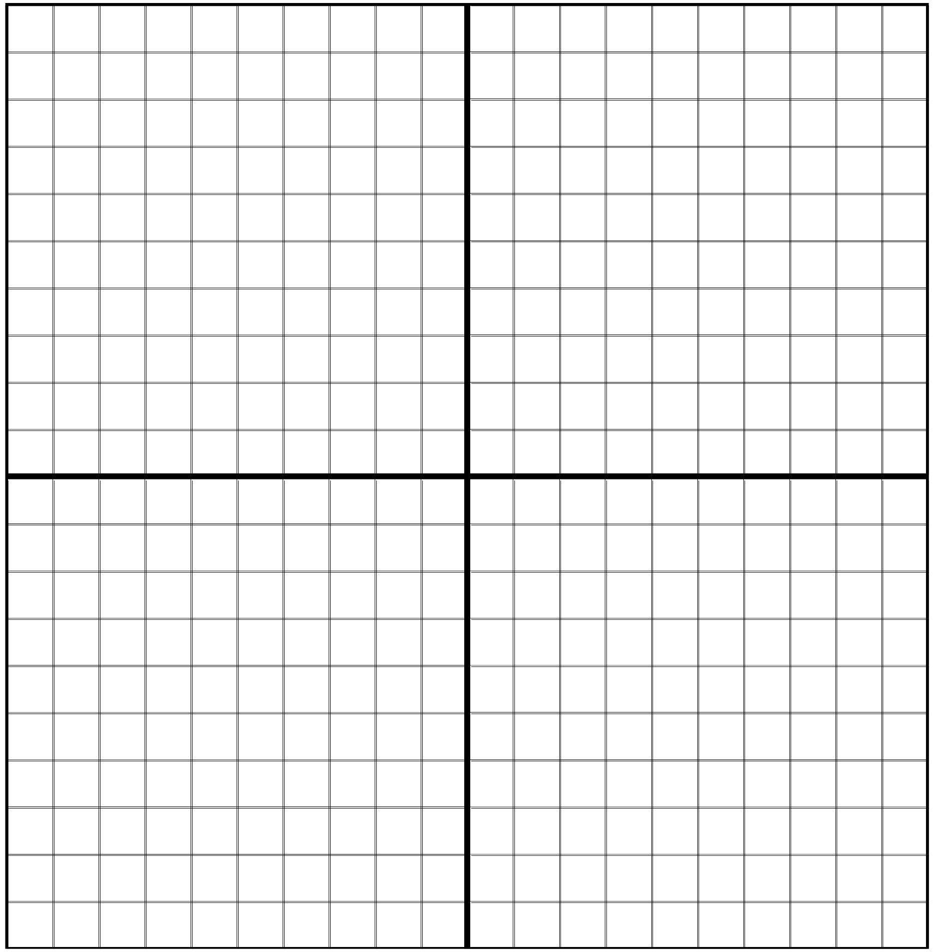
Vertical and Horizontal Translations 5

$$f(x) = 2^x$$

$$f(x) = 2^{(x+4)}$$

$$f(x) = 2^{(x-4)}$$

- Graph. What do you notice? Talk about places where f(x) = 1



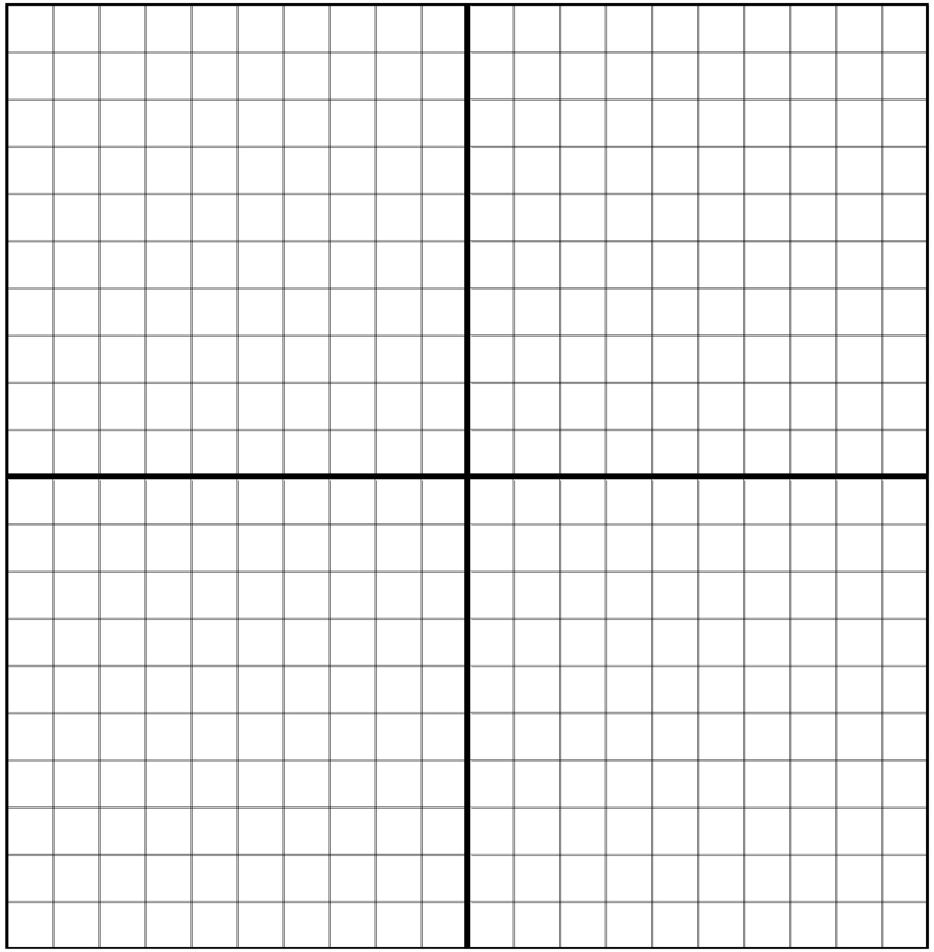
Vertical and Horizontal Translations 6

$$f(x) = 2^{(x+3)}$$

$$f(x) = 2^{x} + 3$$

$$f(x) = 2^{x} - 3$$

- Graph. What do you notice? Talk about the asymptotes.



Make a Conclusion

- Once you've played around and graphed these, help students sort them into two groups (easy for you to see, may be much harder for them).

- Ask them to separate them into which function were of the form

  $f(x \pm c)$ and which were of the form $f(x) \pm c$

- What do they see about how the "c" impacted the movement of the graph?

Formalize the Rule

- Given f(x), what is the rule for $f(x+c)$ and $f(x-c)$?

- Given f(x), what is the rule for f(x) + c and f(x) – c?

- How will they remember this?

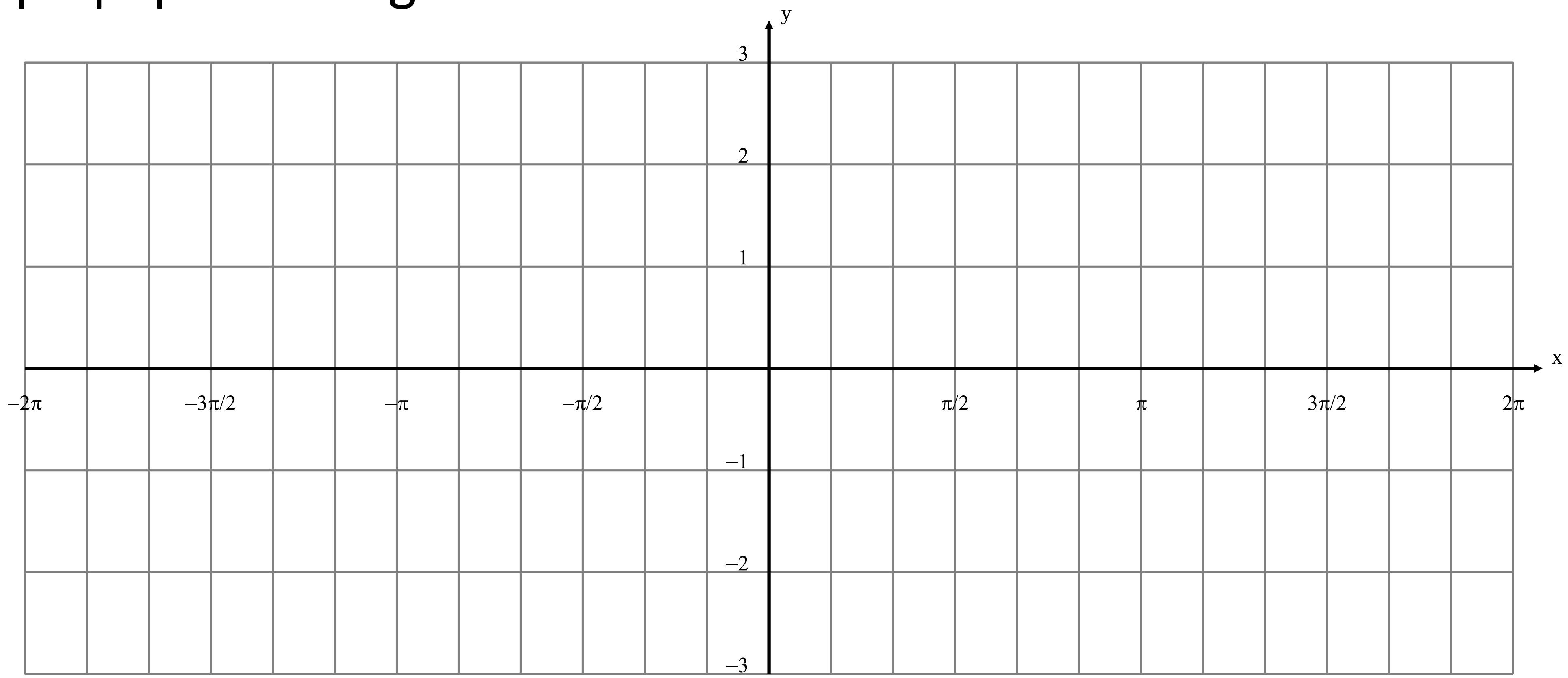
Graphs of Trig Functions

- You can also take this and add trig. functions to the mix, depending when they're taught. Or, repeat this activity when you teach trig.
- The graph paper on the next slide can be used for graphing trig functions because its marked in increments that we usually use for trig functions.
- Use these instructions. You can set the scale on Desmos with the wrench tool on the top right.

Use your calculators for the following problems. Set the window as follows:

Xmin = - 2π **Xmax =** 2π **Xscl =** $\frac{\pi}{2}$ **Ymin = -3 Ymax = 3 Yscl = 1**

- Graph paper for trig. functions:



Translations of Trig. Graphs

- Graph. Describe what happens:

$$y = \sin(x)$$

$$y = \sin\left(x + \frac{\pi}{2}\right)$$

$$y = \sin\left(x - \frac{\pi}{4}\right)$$

$$y = \cos(x)$$

$$y = \cos\left(x - \frac{\pi}{6}\right)$$

$$y = \cos\left(x + \frac{3\pi}{2}\right)$$

$$y = \cos\left(x - \frac{\pi}{2}\right)$$

Reflections

- Graph. What is the difference in the graphs? In the functions?

$$f(x) = (x+2)^2$$

$$g(x) = (-x+2)^2$$

$$h(x) = -(x+2)^2$$

$$m(x) = (x-2)^2$$

But wait – what about cubic functions? Do rules change?

$$f(x) = (x+2)^3$$

$$g(x) = (-x+2)^3$$

$$h(x) = -(x+2)^3$$

$$m(x) = (x-2)^3$$

Try this on Desmos. Then change f(x)

1. $f(x)=x^3$
2. $f(-x)$
3. $-f(x)$
4. $2f(x)$
5. $f(2x)$
6. $-2f(x)$
7. $2f(-x)$
8.

Trig Functions - Reflections

- Graph. Use the window for trig. functions used before:

$$f(x) = \sin x$$
$$g(x) = -\sin x$$
$$p(x) = \sin(-x)$$
$$r(x) = -\sin(-x)$$

$$h(x) = \cos x$$
$$g(x) = -\cos(x)$$
$$m(x) = \cos(-x)$$
$$n(x) = -\cos(-x)$$

- In the first column, demonstrate using Desmos which graphs are the same. Select and deselect using the buttons. Is it the same for sine and for cosine? Why?

Trig Functions - Reflections

- Graph. Use the window for trig. functions used before:

$$f(x) = \sin x$$

$$g(x) = \sin\left(\frac{x}{2}\right)$$

$$p(x) = \frac{1}{2}\sin(x)$$

$$r(x) = 2\sin\left(\frac{1}{2}x\right)$$

$$h(x) = 2\sin x$$

$$g(x) = \sin(2x)$$

$$m(x) = 2\sin(2x)$$

$$n(x) = -2\sin(2x)$$

Reflections

- Try an example that doesn't use trig:

$$f(x) = \sqrt{x}$$

$$f(-x) = \sqrt{-x}$$

$$-f(x) = -\sqrt{x}$$

- Can we develop a rule for these reflections?

- $y = -f(x)$ is the reflection of $y = f(x)$ across the x-axis.
- $y = f(-x)$ is the reflection of $y = f(x)$ across the y-axis

- If (2, 7) is on the graph of f(x), what point is on the graph of

  f(-x) and –f(x)?

- If (3, -4) is on a graph, what is the new point reflected over the x-axis? Over the y-axis?

Cycle Back

- Now that we’ve run through a number of examples, lets go back to the beginning and look at the table of graph transformations.

- If we have time, we can take a look at some additional examples.

Translations

- The same vertical and horizontal translation rules that applied to other functions apply to logs as well.

- Graph. Include the asymptote:

1. $y = \log (x - 2)$
2. $y = \log x + 5$
3. $y = -\log x$
4. $y = \log (x + 3) - 1$
5. $y = \log (-x)$
6. $y = -\log(-x)$

Compare the following graphs

1) $y = 2^x$, $y = 3^x$, $y = 7^x$ $y = 2.5^x$

2) $y = (1/2)^x$ $y = (1/4)^x$ $y = (1/10)^x$

Vertical Translations

- Graph: $y = 2^x + 3$ $y = 2^x - 4$
- Graph: $y = -2^x$ $y = -2^x + 3$ $y = -2^x - 4$

Transforming Exponential Functions

- Compare the following graphs:

$y = 2^x$ $y = 2^{x+3}$ $y = 2^{x-3}$ $y = 2^{x+5}$

$y = 2^x$ $y = 2^{3x}$ $y = 2^{(1/3)x}$

The End

- Thank you for attending today.
- Hopefully you walk away with at least one new idea for your classroom.

- Questions? Frank.Forte@raritanval.edu