# AMTNJ
# 37<sup>th</sup> Annual
# Math Contest
# December 4, 2013

Directions:

➢ ALL answers should be exact unless specified otherwise. In some cases, the desired form of an answer is specified. No other form will be accepted in those cases.
➢ You may only use calculators which are permitted on the SAT I.
➢ You will have exactly 45 minutes to complete this contest. Work quickly, work accurately, and good luck.
➢ You may write on this test paper or on any scrap paper provided by your teacher, but your answers must be written on the Student Response Sheet, to be official.

1) A gym class has "x" students. After "y" teams are formed of "z" players each, what percent of the students have yet to be chosen?

2) How many different triangles have sides whose lengths are integers, if the longest side is 6?      (more than one side can have a length of 6)

3) What is the units digit of $13^{13}$?

4) If the median of a list of 2013 consecutive even integers is 0, what is the least number?

5) How many four digit numbers are rising? (a rising number is a number in which each digit to the right is larger than its preceding digit)

6) If $log_7(x + 4) = 1 - log_7(x - 2)$ , what is the value of x?

7) Given $\triangle ABC$ with median CM=27. If a perpendicular is drawn from point B to $\overrightarrow{CM}$ extended, intersecting it at a point R and BR=4. Find the area of $\triangle ABC$ ?

8) Find f(4) if, for all real $x \neq 2$, if the function f(x) satisfies:

$$\frac{2f(x) - f(1-x)}{x-2} = 2x - 1$$

9) The roots of $z^4 + az^3 + bz^2 + cz + 62500 = 0$ are $x \pm yi \ and \ y \pm xi$, where x and y are positive integers and $x < y$. Find all possible pairs of values for x and y.

10)    In parallelogram ABCD, points X and Y trisect sides CD and AB respectively, such that $AY = \frac{1}{2}YB$ and $XY \parallel BC$.   If the area of ABCD is 180 square units and point Z is the midpoint of segment XY, what is the area of $\triangle AYZ$ ?

11)    Find the value(s) of x such that:     $\binom{2013}{1006} + \binom{2013}{1005} = \binom{2014}{x}$ .

Given that $\binom{n}{r} = \frac{n!}{r!(n-r)!}$ .

12) In a cryptarithm each letter stands for a different digit.  Solve the following cyrptarithm and state the value of the letter "D":  $SEND + MORE = MONEY$,

13)    If the perimeter of an isosceles right triangle is 18mm, what is its exact area?

14)    Find the exact value of $\tan(15°)$ in simplified rationalized form.

15)    Caffeine is in medications, foods, and energy supplements.  After 8 hours, ¾ dose of caffeine is gone.  Exactly how much caffeine will be left after 12 hours?  You must assume that caffeine absorption is an exponential function.