AMTNJ
38th Annual
Math Contest
December 10, 2014

Directions:

➢ Answers should only be in the form specified: decimals must be at least three decimal places rounded or truncated. For example, 2/3 = 0.666 or 0.667. Fractions and irrational quantities must be in simplest form. In some cases, the desired form of an answer is specified. No other form will be accepted in those cases.

➢ You may only use calculators which are permitted on the SAT I.

➢ You will have exactly 45 minutes to complete this contest. Work quickly, work accurately, and good luck.

➢ You may write on this test paper or on any scrap paper provided by your teacher, but your answers must be written on the Student Response Sheet, to be official.

1) How many times does the digit 6 appear in all the integers between 600 and 699, inclusive?

2) The sides of triangle CAT are 10, 10, and 12 while the sides of triangle DOG are 10, 10, and 16. What is the exact difference in the areas of the two triangles?

3) Find the exact length of the chord formed when the line $x - y = -6$ intersects the parabola $x^2 + y = 12$?

4) Given that the roots of $3x^2 + 8x - 3 = 0$ are m and n and that the roots of $px^2 + qx + r = 0$, are m^2 and n^2, what is the value of $p + q + r =$, given that p, q, and r are integers?

5) The right triangle with sides 7, 24, and 25 is inscribed in a circle. Find the area between the triangle and circle to 2 decimal places.

6) A 4 digit number N is formed by a random permutation of the digits in 2014. What is the probability that N is prime?

7) The roots of the equation $x^3 - 24x^2 + 183x - 440 = 0$ are the first 3 terms of an arithmetic sequence with a positive difference. Find the 10th term of the sequence.

8) Given quadrilateral ABCD with diagonals AC and BD perpendicular to each other. If AC = x and BD = y, what is the area of ABCD?

9) The arithmetic mean of a set of 50 numbers is A. If the numbers 63 and 37 are removed from the set of numbers, what is the arithmetic mean of the remaining numbers?

10)   Rewrite the following as a single expression in
simplest form: $a * log_c b - b * log_c a =$

11)   Given triangle ABC with points E and F the midpoints
of sides AB and BC respectively. Point X lies on segment
EF, 2/3 of the way from point E to point F. If the area of
triangle ABC is Z, what is the area of triangle AXC?

12)   For what value of k, will x+2 be a factor of:
$x^3 + 5x^2 + kx + 9$?

13)   What is the ratio of the area of a square inscribed in a
semicircle of radius r to the area of a square inscribed in a
circle of radius r?

14)   Find the positive integer value of x in:

$$\frac{12!}{5!7!} + \frac{12!}{6!6!} = \frac{x!}{6!7!}$$

15)   Given:
$cos(x) = cos(47°) cos(13°) - sin(47°) sin(13°)$,
find the exact value of sin(2x)=