

**Directions:**

- Your answers should be in the form specified in the problem. Approximate answers must be at least three decimal places rounded or truncated (ex:  $\frac{2}{3} \approx 0.666$  or  $0.667$ ), and exact answers must be in simplest form (ex:  $\frac{5}{15}$  will not be accepted for  $\frac{1}{3}$ , and  $\sqrt[3]{48}$  will not be accepted for  $2\sqrt[3]{6}$ ). When the desired form is specified in a problem, any other form of the answer will not receive credit.
  - You may only use calculators that are permitted on the SAT I.
  - You may write on this contest and use additional paper you receive from your teacher, but you should write your answers on the **Individual Student Cover Page** to be official and receive credit.
  - You will have exactly 45 minutes to complete the problems in this contest. Work quickly and with accuracy.
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**Problems:**

1. How many ordered pairs  $(a, b)$  of non-negative integers are there that satisfy  $5a + 4b \leq 20$  ?
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2. Find the smallest positive integer that is divisible by every positive integer less than 10.
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3. Points  $C$  and  $D$  are on the line segment  $\overline{AB}$  of fixed length, with  $D$  between  $C$  and  $B$  as shown. Note: The diagram is not drawn to scale.



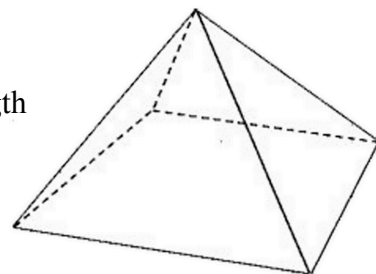
If  $\frac{AC}{CB} = \frac{1}{3}$  and  $\frac{CD}{DB} = \frac{2}{5}$ , what is the value of the ratio  $\frac{CD}{AB}$  ?

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4. Find the coordinates of the point on the line  $y = 2x + 3$  that is equidistant from, and closest to the two points  $(14, 1)$  and  $(-10, 13)$ .
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5. The amount in grams of radioactive isotope bromine-77 remaining after  $t$  days is modeled by the exponential function  $A(t) = A_0(0.5)^{\frac{t}{57}}$ , where  $A_0$  is the amount in grams of the isotope present at  $t = 0$ . What is the daily decay rate of bromine-77? Write your answer as a percent rounded to the nearest tenth.
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6. The base of a rectangular pyramid is 26 square units, and its volume is 54 cubic units. If the width of the base is increased by 20%, the length decreased by 15%, and the height increased by 5%, find the volume of the new pyramid.

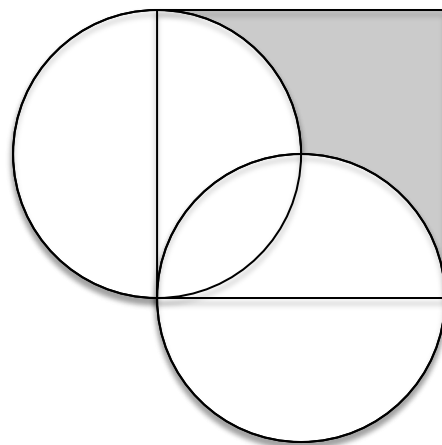


7. Let  $x$  and  $y$  be positive real numbers. If  $x = 3 + \frac{1}{3 + \frac{1}{x}}$  and  $y = 3 + \frac{1}{3 + \frac{1}{y}}$ ,

find the exact value of  $x + y$ .

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8. The side of the square shown on the right has length 6. The two circles are tangent to the two sides of the squares, and their diameters coincide with the other two sides. Find the area of the shaded region that is inside the square but lies outside the two circles.



9. Fifteen students from the environmental club at a high school volunteered to clean a local park. The club advisor would like to have three teams of five students each. If each team is to be assigned to a different section of the park, in how many different ways can the club advisor assign the 15 students to the three groups?
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10. What is the exact value of  $k$  for which  $(x + 2)$  is a factor of  $4x^4 - (kx)^3 - 8x - 8$ ?
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11. A *palindrome* is a number that reads the same from left to right and from right to left. 2, 22 and 232 are examples of palindromes. How many palindromes between 1 and 1000 are divisible by 11?
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12. A circle with radius 2 units rolls along the outside of a regular hexagon of side 4 units. Find the exact value of the total distance traveled by the center of this circle after it completes its trip along the outside of the hexagon exactly once, and returns to its starting position.
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13. Let  $v$  and  $w$  be non-negative integers less than or equal to 10. For how many ordered pairs  $(v, w)$  is the expression  $(v + w)^2 + (vw - 1)^2$  a prime number?
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14. Let the sequence  $a_n$  have the property that, for any nonnegative integer  $k$ ,  $\sum_{i=4k+1}^{4(k+1)} a_i = 2015$ .

Evaluate  $\sum_{i=1}^{2016} a_i$ .

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15. Given the rectangle  $ABCD$  in which  $AB = 10$  and  $BC = 12$ . If  $M$  is the midpoint of side  $CD$  and  $K$  is a point on  $AM$  such that  $AK = AD$ , what is the area of quadrilateral  $ABCK$ ?
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