

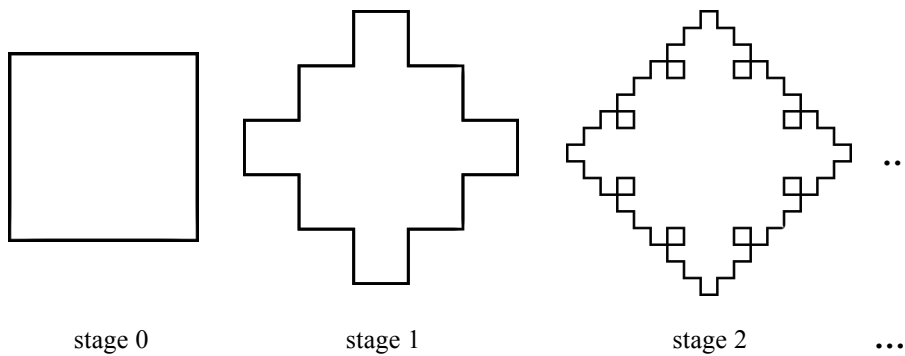
**Directions:**

- Your answers should be in the form specified in the problem. Approximate answers must be at least three decimal places rounded or truncated (ex:  $\frac{2}{3} \approx 0.666$  or  $0.667$ ), and exact answers must be in simplest form (ex:  $\frac{5}{15}$  will not be accepted for  $\frac{1}{3}$ , and  $\sqrt[3]{48}$  will not be accepted for  $2\sqrt[3]{6}$ ). When the desired form is specified in a problem, any other form of the answer will not receive credit.
- You may only use calculators that are permitted on the SAT.
- You may write on this contest and use additional paper you receive from your teacher, but you should write your answers on the **Individual Student Cover Page** to be official and receive credit.
- You will have exactly 45 minutes to complete the problems in this contest. Work quickly and with accuracy.

**Problems:**

1. Find the exact value of  $\frac{2^{2020} - 2^{2016}}{2^{2020} + 2^{2017}}$ . Write your answer in simplest form.

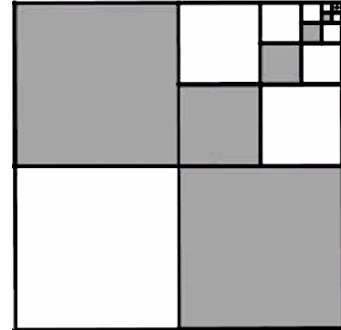
2. In the sequence of shapes shown below, how many sides does the shape in stage 4 have?



3.  $f(x) = \frac{1}{1 - \frac{1}{1 - \frac{1}{x}}}$ . Find  $|f(2) - f(4) + f(6)|$ .

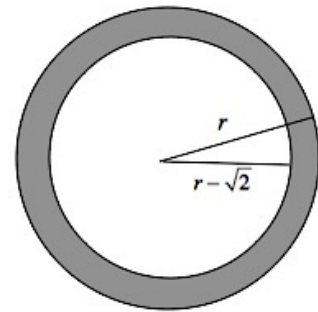
4. Given the points  $M(0, 220)$  and  $N(n, 2020)$ , where  $n$  is a positive integer. For how many values of  $n$  is the slope of  $MN$  an integer?
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5. If the shading pattern in the top right quarter of the square continues indefinitely, what fraction of the square is shaded?



6. Let  $f(x) = 2 - 3x$  and  $g(x) = \frac{1}{x+3}$ . Find the **exact value** of  $g(f(g(23)))$ .
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7. The shaded area between two concentric circles with radii  $r$  and  $r - \sqrt{2}$  is  $2\pi + 6\pi\sqrt{2}$  square units. When the sum of the radii is written as  $a + \sqrt{b}$ , find  $a + b$ .

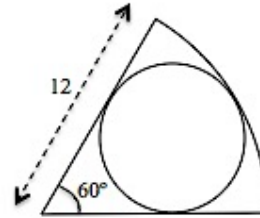


8. The faces of two fair eight-sided dice are numbered 1 to 8. What is the probability of getting a sum of 8 when you roll the two dice once?
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9. A robot needs to get from  $(0, 0)$  to  $(9, 11)$  on a gridded level field. The robot needs to avoid a square block that is on the field with its vertices at  $(2, 2)$ ,  $(6, 2)$ ,  $(2, 6)$  and  $(6, 6)$ . If the robot can move freely in any direction on the field, find the shortest distance the robot can travel to reach its destination.
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10. For what value of  $m$  do the real solutions of  $x^2 - 8x + m = 0$  differ by 5?
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11. A circle is inscribed in a sector of another circle with a radius of 12 units and a central angle of  $60^\circ$ . What is the area of the inscribed circle?



12. A regular hexagon and a circle have the same area. What is the ratio of the perimeter of the hexagon to the circumference of the circle?

13. Points  $X$  and  $Y$  are on a line segment  $\overline{WZ}$ , with  $Y$  between  $X$  and  $Z$  as shown below.

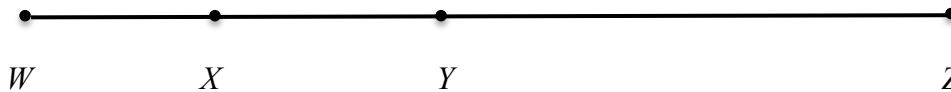


Figure not drawn to scale

If  $\frac{WX}{XZ} = \frac{1}{5}$  and  $\frac{XY}{YZ} = \frac{2}{3}$ , what is the value of  $\frac{XY}{WZ}$ ?

14. What is the 2020<sup>th</sup> digit after the decimal in the expansion of  $\frac{1}{41}$ ?

15. In a certain type of  $3 \times 3$  array, the entries can be 0 or 1 in such a way that there is no more than one 0 in each row and each column.

For example, the array  $\begin{array}{ccc} 0 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 0 \end{array}$  is acceptable, whereas  $\begin{array}{ccc} 0 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{array}$  is not acceptable.

How many acceptable  $3 \times 3$  arrays are possible?

