

Directions:

- Your answers should be in the form specified in the problem to receive credit. Approximate answers must be at least three decimal places rounded or truncated (ex: $\frac{2}{3} \approx 0.666$ or 0.667), and exact answers must be in simplest form (ex: $\frac{5}{15}$ will not be accepted for $\frac{1}{3}$, and $\sqrt[3]{48}$ will not be accepted for $2\sqrt[3]{6}$).
- Only **scientific calculators** are allowed on this contest.
- Do **NOT** use calculators that:
 - can access the internet,
 - can communicate with other devices,
 - store programs, formulas, or notes,
 - use a computer algebra system
 - have dynamic geometry software
- You may write on this contest and use additional paper you receive from your teacher, but you should write your answers on the **Individual Student Cover Page** to be official and receive credit.
- You will have exactly 45 minutes to complete the 9 problems in this contest. Work carefully and with accuracy.

Problems:

1. What is the units digit of the product $(2^2 - 1)(2^4 - 1)(2^6 - 1) \dots (2^{2022} - 1)$?

2. Given points $S(1, 7)$, $U(x, 4)$ and $M(-4, -7)$.
Find the exact value of x that will minimize the sum $SU + UM$.

3. Let $f(x) = 2x^3 + 3x^2 + cx + d$ where c and d are constants.
If 2 and -3 are real zeros of f , find the third zero.

4. A circle with radius r along with two regular hexagons are shown. One hexagon is inscribed in the circle and the other hexagon circumscribes the circle. Find the radius of the circle for which the sum of the areas of the two hexagons is $224\sqrt{3} \text{ cm}^3$.

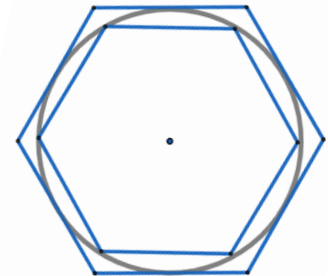


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5. Find the exact value of the continued fraction $4 + \frac{3}{4 + \frac{3}{4 + \frac{3}{4 + \dots}}}$

6. The square $ABCD$ is inscribed in right triangle TRI . Side \overline{DC} is along the hypotenuse \overline{TI} , vertex A is along \overline{TR} , and vertex B is along \overline{RI} , as shown in the figure.

If $TD = 45$ and $CI = 20$, what is the area of square $ABCD$?

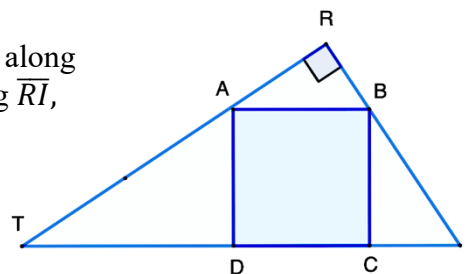


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7. Five identical circles of radius 5 centimeters are arranged around a smaller circle, as shown in the figure. Each outer circle is tangent to two adjacent circles of radius 5 as well as the smaller circle in the center. The smaller circle in the center is tangent to all five larger circles. A band is tightly wrapped around the outside of the five circles. Find the exact length of the band.

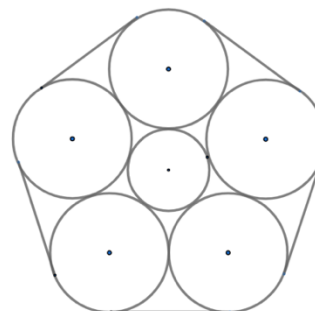


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8. At 20,310 feet (≈ 3.847 miles), Denali in Alaska is the highest mountain in North America. Given that the radius of the Earth is about 3,959 miles, Find the distance (in miles) from the summit of Denali to the furthest visible point along the horizon.

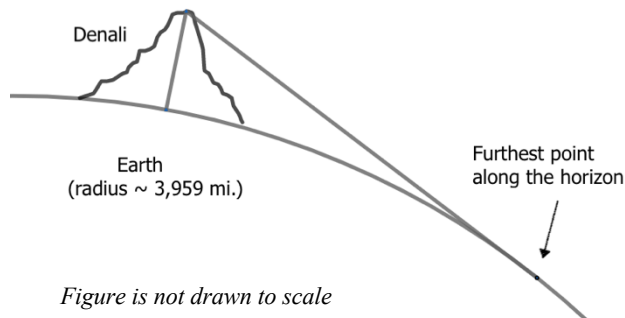


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9. The table below shows the percentage of party affiliations among all registered voters in a local municipality and the percentage within each party who voted during a recent local election.

	Party A	Party B	Party C
Party Affiliation	30%	50%	20%
% Who Voted	65%	82%	50%

Given that a randomly selected person from the municipality has voted in the election, what is the probability that she is a member of party B?

