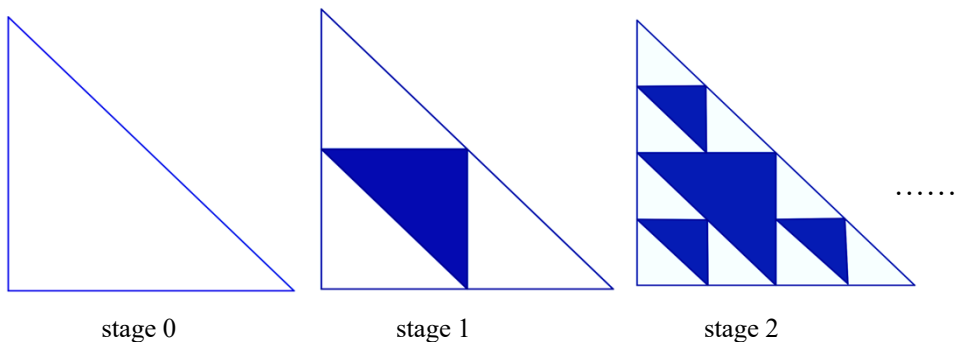


**Directions:**

- Your answers should be in the form specified in the problem to receive credit. Approximate answers must be at least three decimal places rounded or truncated (ex:  $\frac{2}{3} \approx 0.666$  or  $0.667$ ), and exact answers must be in simplest form (ex:  $\frac{5}{15}$  will not be accepted for  $\frac{1}{3}$ , and  $\sqrt[3]{48}$  will not be accepted for  $2\sqrt[3]{6}$ ).
- Only **scientific calculators** are allowed on this contest.
- Do **NOT** use calculators that:
  - can access the internet,
  - can communicate with other devices,
  - store programs, formulas, or notes,
  - use a computer algebra system
  - have dynamic geometry software
- You may write on this contest and use additional paper you receive from your teacher, but you should write your answers on the **Individual Student Cover Page** to be official and receive credit.
- You will have exactly 45 minutes to complete the 9 problems in this contest. Work carefully and with accuracy.

**Problems:**

1. Let  $f(x) = x^2 + ax + b$ . If  $f(-3) + f(3) = 0$ , find  $f(-5) + f(5)$ .
2. If the longest side of a right triangle is  $10^{2023} + 1$  units, and the other sides are  $10^{2023} - 1$  units and  $n10^m$  units, find the value of  $n \times m$ .
3. In the sequence of triangles shown below, stage 0 has one triangle and stage 1 has five triangles. If the pattern continues, how many triangles will stage 4 have?



4. A function  $f$  has zeros at  $3$ ,  $\frac{5}{8}$  and  $-\frac{2}{3}$ . If  $g(x) = -3f\left(-\frac{x}{2}\right)$ , what is the sum of the zeros of  $g$ ?  
Write your answer in exact form.

5. Water in a large cylindrical tank is 100 inches deep. When a cylinder with a smaller base is placed in the tank, the water level rises to 120 inches, as shown on the right.

If the large tank has radius  $R$ , and the smaller cylinder has radius  $r$ , find the exact value of  $\frac{R}{r}$ .

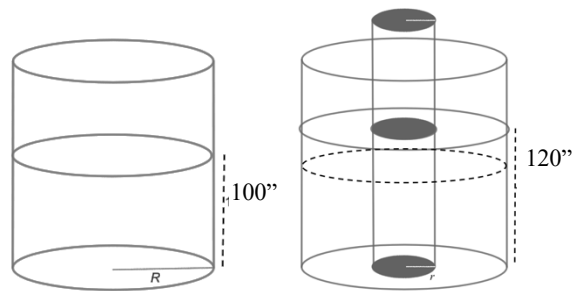


Figure is not drawn to scale

6. Equilateral triangle  $PQR$  is inside another equilateral triangle  $ABC$ , with  $P$ ,  $Q$  and  $R$  along sides  $AB$ ,  $BC$  and  $CA$  respectively, one unit away from each vertex as shown in the figure to the right.

If  $AB = BC = CA = 6$ , find the area of triangle  $PQR$ .

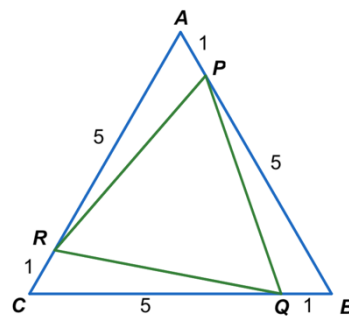


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7. In a lottery game at the local fair, a player chooses four distinct numbers between 1 and 25 for the chance to win \$10,000. To win, your 4 numbers must match the 4 randomly chosen numbers at the drawing. Each play costs \$3, and you can play this game multiple times.

What is the least amount you must spend to guarantee a win?

Play for \$3				
Pick 4 for a chance to win \$10,000				
1	2	3	4	5
6	7	8	9	10
11	12	13	14	15
16	17	18	19	20
21	22	23	24	25

8. Two lines that intersect at  $I$  form  $120^\circ$ , and the two circles with radii 2 and 3 are tangent to the lines at points  $F$ ,  $G$ ,  $H$  and  $K$  as shown in the figure on the right.

What is the area of the shaded region?

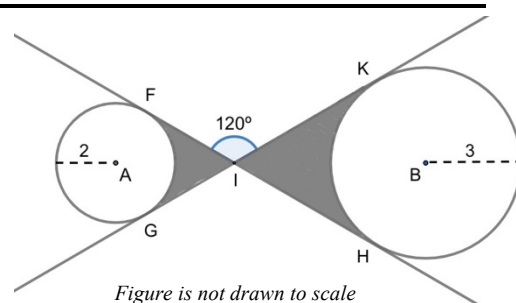


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9. The  $n^{\text{th}}$  term  $a_n$  of a sequence of numbers  $a_1, a_2, a_3, \dots$  is defined by  $a_n = a_{n-1} + a_{n-2}$ ,

where  $a_1 = 1$  and  $a_2 = 2$ .

Find  $a_1 + a_2 + a_3 + \dots + a_7$ , the sum of the first 7 terms of this sequence.

