AMTNJ High School Contest 47 Problems January 22, 2025

Directions:

- Your answers should be in the form specified in the problem to receive credit. Approximate answers must be at least three decimal places rounded or truncated (ex: $\frac{2}{3} \approx 0.666\ or\ 0.667$), and exact answers must be in simplest form (ex: $\frac{5}{15}$ will not be accepted for $\frac{1}{3}$, and $\sqrt[3]{48}$ will not be accepted for $2\sqrt[3]{6}$).

- Only **scientific calculators** are allowed on this contest.

- Do **NOT** use calculators that**:**
  can access the internet,
  can communicate with other devices,
  store programs, formulas, or notes,
  use a computer algebra system
  have dynamic geometry software

- You may write on this contest and use additional paper you receive from your teacher, but you should write your answers on the **Individual Student Cover Page** to be official and receive credit.

- You will have exactly 45 minutes to complete the 9 problems in this contest.
  Work carefully and with accuracy.

Problems:

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1. If a<b, then $3^2 + 4^2 + 5^2 + 12^2 = a^2 + b^2$ is satisfied by only one pair of positive integers (a,b). what is the value of a + b?

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2. This coming Halloween, Tom plans to scare twice as many people as Sam, and Sam plans to scare three times as many people as Roz. In all, they plan to scare at most 2025 people. If no one is scared more than once, at most how many people does Sam plan to scare?

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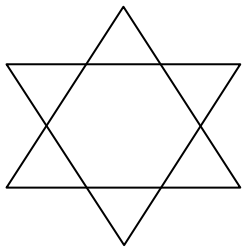
3. When two congruent equilateral triangles share a common center, their union can be a star, as shown. If their overlap is a regular hexagon with an area of 60, what is the area of one of the original equilateral triangles?



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4. Between 1 and 200, there is a sequence of 13 consecutive integers, none of which is prime. What is the sum of these integers, and what is the smallest of these integers?

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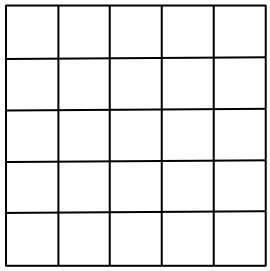
5. Counting every possible square of each size from 1x1 to 5x5, what is the total number of distinct squares that can be traced out along the lines of the accompanying grid?



6. The four numbers a<b<c<d can be paired in six different ways. If each pair has a different sum, and if the four smallest sums are 1,2,3, and 4, what are all possible values of d?

7. Let ℓ_1 be the perimeter of the ellipse $\frac{x^2}{36} + \frac{y^2}{16} = 1$ and ℓ_2 be the perimeter of the ellipse $\frac{x^2}{36} + \frac{y^2}{81} = 1$. What is the ratio of ℓ_1 to ℓ_2

8. If (a,2025) and (2025,b) are different points on the graph of the line $y = \frac{3}{4}x + 5$, what is the value of $\frac{2025-a}{2025-b}$?

9. What are all the ordered pairs of (a,b), with 0 < a < b, for which $(\sqrt[a]{2025})(\sqrt[b]{2025}) = \sqrt[9]{2025}$?